CSE 311: Foundations of Computing I

Homework 3 (due Wednesday, April 18th at 11:59 PM)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use results from lecture, the theorems handout, and previous homeworks without proof.*

1. Not So Negative (18 points)

For each of the following English statements, (i) translate it into predicate logic, (ii) write the negation of that statement in predicate logic with the negation symbols pushed as far in as possible so that any negation symbols is directly in front of a predicate, and then (iii) translate the result of (ii) back to English (natural if possible).

For the logic, let your domain of discourse be people and activities. You should use only the predicates Loves(x, y), Likes(x, y), and Hates(x, y), which say that person x loves, likes, or hates (respectively) activity y; the predicates Person(x) and Activity(x), which say whether x is a person or activity (respectively); and the predicate Equal(x, y), which says whether x and y are the same object.

(a) [6 Points] Fred likes some activity other than hiking.

(b) [6 Points] There is someone who doesn't love any activity but likes every activity.

(c) [6 Points] Everyone who likes hiking and swimming has an activity that they love.

2. Formal Proofs (25 points)

(a) [15 Points] Write a formal proof using inference rules that given $(p \wedge \neg q) \vee (\neg p \wedge q)$, $r \rightarrow \neg s$, and $(s \wedge p) \rightarrow r$, the proposition $s \rightarrow q$ must also be true.

(b) [10 Points] Write a formal proof using inference rules of $((p \rightarrow q) \wedge (r \rightarrow \neg q)) \rightarrow (r \rightarrow \neg p)$

3. Spoofclusions (17 points)

Theorem: Given $s \rightarrow (p \wedge q)$, $\neg s \rightarrow r$, and $(r \vee p) \rightarrow q$, prove q.

"Spoof:"

1.	$\neg s \rightarrow r$		Given
2.	$(r \vee p) \rightarrow q$		Given
3.	$r \rightarrow q$		$\vee$ Elim: 2
	4.1. $\neg s$	Assumption	
	4.2. r	MP: 4.1, 1	
	4.3. q	MP: 4.2, 3	
4.	$\neg s \rightarrow q$		Direct Proof Rule
	5.1. s	Assumption	
	5.2. $s \rightarrow (p \wedge q)$	Given	
	5.3. $p \wedge q$	MP: 5.1, 5.2	
	5.4. q	$\wedge$ Elim: 5.3	
5.	$s \rightarrow q$		[Direct Proof Rule]
6.	$(s \rightarrow q) \wedge (\neg s \rightarrow q)$		$\wedge$ Intro: 4, 5
7.	$(\neg s \vee q) \wedge (\neg\neg s \vee q)$		$\rightarrow$ Elim: 6
8.	$(\neg s \vee q) \wedge (s \vee q)$		Double Negation
9.	$((\neg s \vee q) \wedge s) \vee ((\neg s \vee q) \wedge q)$		Distributivity
10.	$((\neg s \vee q) \wedge s) \vee (q \wedge (\neg s \vee q))$		Commutativity
11.	$((\neg s \vee q) \wedge s) \vee (q \wedge (q \vee \neg s))$		Commutativity
12.	$((\neg s \vee q) \wedge s) \vee q$		Absorption
13.	$(s \wedge (\neg s \vee q)) \vee q$		Commutativity
14.	$((s \wedge \neg s) \vee q) \vee q$		Associativity
15.	$(F \vee q) \vee q$		Negation
16.	$(q \vee F) \vee q$		Commutativity
17.	$q \vee q$		Identity
18.	q		Idempotence

(a) [6 Points] There are two errors in this proof. Indicate which lines contain the errors and, for each one, explain (as briefly as possible) why that line is incorrect.

(b) [5 Points] Is the conclusion of the "spoof" correct? Explain why or why not.

(c) [6 Points] Give a correct proof of what is claimed in lines 6–18, i.e., that, from $(s \rightarrow q) \wedge (\neg s \rightarrow q)$, we can infer that q is true.

4. Mind Your P's and Q's (20 points)

Using the logical inference rules and equivalences we have given, write a *formal* proof that given $\forall x\ (\exists y\ P(x, y) \rightarrow \neg Q(x))$, $\forall x\ (\neg R(x) \rightarrow (Q(x) \vee \neg P(x, x))$, and $\exists x\ P(x, x)$, you can conclude that $\exists x\ R(x)$.

5. Hip to be square (20 points)

We say that an integer n is a square iff there exists a $k \in \mathbb{Z}$ such that $n = k^2$.

(a) [10 Points] Give a formal proof that, if integers n and m are squares, then nm is a square. In addition to the inference rules discussed in class, you can also rewrite an algebraic expression to equivalent ones using the rule "Algebra".

(b) [10 Points] Write your proof from part (a) as an English proof.

6. EXTRA CREDIT: Aarh! Me Hearties (-NoValue- points)

Five pirates, called Ann, Brenda, Carla, Danielle and Emily, found a treasure of 100 gold coins.
On their ship, they decide to split the coins using the following scheme.

- The first pirate in alphabetical order becomes the chief pirate.
- The chief proposes how to share the coins, and all other pirates (excluding the chief) vote for or against it.
- If 50% or more of the pirates vote for it, then the coins will be shared that way.
- Otherwise, the chief will be thrown overboard, and the process is repeated with the pirates that remain.

Thus, in the first round Ann is the chief: if her proposal is rejected, she is thrown overboard and Brenda becomes the chief, etc; if Ann, Brenda, Carla, and Danielle are thrown overboard, then Emily becomes the chief and keeps the entire treasure.
The pirates' first priority is to stay alive: they will act in such a way as to avoid death. If they can stay alive, they want to get as many coins as possible. Finally, they are a blood-thirsty bunch, if a pirate would get the same number of coins if she voted for or against a proposal, she will vote against so that the pirate who proposed the plan will be thrown overboard.
Assuming that all 5 pirates are intelligent (and aware that all the other pirates are just as aware, intelligent, and bloodthirsty), what will happen? Your solution should indicate which pirates die, and how many coins each of the remaining pirates receives.