

CSE 311: Foundations of Computing I

Section 7: Strong Induction and Recursive Sets

1. Binary Representations

Prove that every natural number can be written as a sum of *distinct* powers of two. (I.e., that it has a unique binary representation.)

2. Cantelli's Rabbits

Xavier Cantelli owns some rabbits. The number of rabbits he has in a given year is described by the function f :

$$\begin{aligned}f(0) &= 0 \\f(1) &= 1 \\f(n) &= 2f(n-1) - f(n-2) \quad \text{for } n \geq 2\end{aligned}$$

Determine, with proof, the number, $f(n)$, of rabbits that Cantelli owns in year n .

3. Recursively Defined Sets of Strings

For each of the following, write a recursive definition of the sets satisfying the following properties. Briefly justify that your solution is correct.

- (a) Binary strings of even length.
- (b) Binary strings not containing 10 as a substring and having at least as many 1s as 0s.