CSE 311: Foundations of Computing I

Section 6: Induction

1. Extended Euclidean Algorithm

(a) Find the multiplicative inverse y of $7 \bmod 33$. That is, find y such that $7y \equiv 1 \pmod{33}$. You should use the extended Euclidean Algorithm. Your answer should be in the range $0 \le y < 33$.

(b) Now, solve $7z \equiv 2(\text{mod } 33)$.

2. Induction with Sums: Equality

For any $n \in \mathbb{N}$, define S_n to be the sum of the squares of the first n positive integers, or

$$S_n = \sum_{i=1}^{n} i^2.$$

For all $n \in \mathbb{N}$, prove that $S_n = \frac{1}{6}n(n+1)(2n+1)$.

3. A Strict Inequality

Prove that $6n + 6 < 2^n$ for all $n \ge 6$.

4. Divisibility by Induction

Prove that $9 \mid n^3 + (n+1)^3 + (n+2)^3$ for all $n > 1$ by induction.

5. Another Inequality

Prove for all $n \in \mathbb{N}$ that, if you have numbers $a_1, \cdots, a_n$ and $b_1, \cdots, b_n$, with $\forall i \in [n].\ a_i \le b_i$, then:

$$\sum_{i=1}^{n} a_i \le \sum_{i=1}^{n} b_i$$