CSE 311: Foundations of Computing I

Section 5: Number Theory

1. Modular Arithmetic

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

(b) Prove that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{n}$.

2. Casting Out Nines

Let $n \in \mathbb{N}$. Prove that if $n \equiv 0 \pmod{9}$, then the sum of the digits of n is a multiple of 9.
You may use without proof that $a \equiv b \pmod{m} \rightarrow a^i \equiv b^i \pmod{m}$.

3. Perfect Squares

Prove that if $n^2 + 1$ is a perfect square, where n is an integer, then n is even.

4. Divisors and Primes

Prove that if n is a positive integer such that the sum of the divisors of n is $n + 1$, then n is prime.