CSE 311: Foundations of Computing I

Section 3: Predicate Logic and Inference Solutions

1. Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other (or if they are equivalent).

(a) $\forall x\, \forall y\, P(x, y)$ $\qquad\qquad$ $\forall y\, \forall x\, P(x, y)$

Solution:

These sentences are the same; switching universal quantifiers makes no difference.

(b) $\exists x\, \exists y\, P(x, y)$ $\qquad\qquad$ $\exists y\, \exists x\, P(x, y)$

Solution:

These sentences are the same; switching existential quantifiers makes no difference.

(c) $\forall x\, \exists y\, P(x, y)$ $\qquad\qquad$ $\forall y\, \exists x\, P(x, y)$

Solution:

These are only the same if P is symmetric (e.g. the order of the arguments doesn't matter). If the order of the arguments does matter, then these are different statements. For instance, if $P(x, y)$ is "$x < y$, then the first statement says "for every x, there is a corresponding y such that $x < y$, whereas the second says "for every y, there is a corresponding x such that $x < y$. In other words, in the first statement y is a function of x, and in the second x is a function of y.

(d) $\forall x\, \exists y\, P(x, y)$ $\qquad\qquad$ $\exists x\, \forall y\, P(x, y)$

Solution:

These two statements are usually different.

2. Formal Proof (Direct Proof Rule)

Show that $\neg p \to s$ follows from $p \vee q$, $q \to r$ and $r \to s$.

Solution:

1.	$p \vee q$	[Given]
2.	$q \to r$	[Given]
3.	$r \to s$	[Given]
4.1.	$\neg p$	[Assumption]
4.2.	q	[Elim of $\vee$: 1, 4.1]
4.3.	r	[MP of 4.2, 2]
4.4.	s	[MP 4.3, 3]
4.	$\neg p \to s$	[Direct Proof Rule]

3. Formal Proof

Show that $\neg p$ follows from $\neg(\neg r \vee t)$, $\neg q \vee \neg s$ and $(p \to q) \wedge (r \to s)$.

Solution:

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \to q) \wedge (r \to s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law, 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation, 5]
7.	$r \to s$	[Elim of $\wedge$, 3]
8.	s	[MP, 6,7]
9.	$\neg q$	[Elim of $\vee$, 2, 8]
10.	$p \to q$	[Elim of $\wedge$, 3]
11.	$\neg q \to \neg p$	[Contrapositive, 10]
12.	$\neg p$	[MP, 9,11]