CSE 311 Lecture 26: Limitations of DFAs, NFAs, and Regular Expressions

Emina Torlak and Kevin Zatloukal

Topics

From NFAs to DFAs

A quick wrap-up of Lecture 25.

DFAs ≡ NFAs ≡ regular expressions

They are all the same and represent *regular languages*.

Languages and representations

Regular, context-free, and other languages.

Proving irregularity

A proof template for showing that a language is not regular.

From NFAs to DFAs

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NFAs and DFAs

Every DFA is an NFA.

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We'll see how to construct the start state, remaining states and transitions, and the final states of the DFA.

NFAs to DFAs: the start state

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All states reachable from the start state of the NFA using only ε edges.

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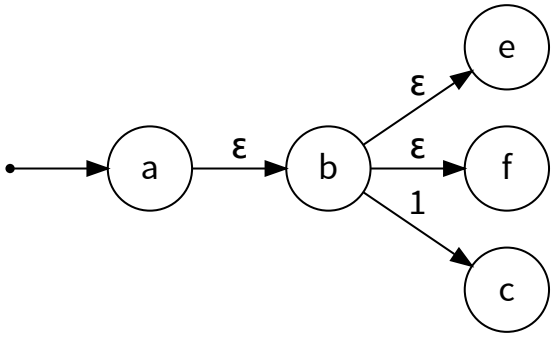

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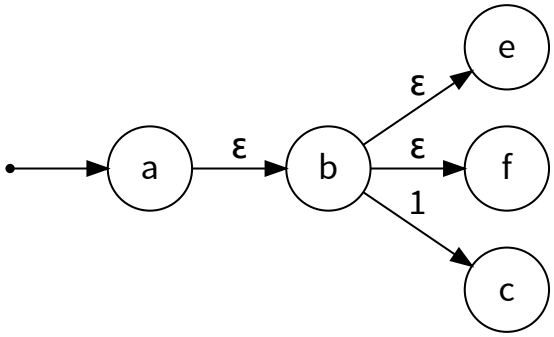

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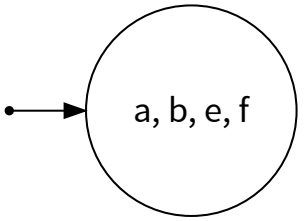

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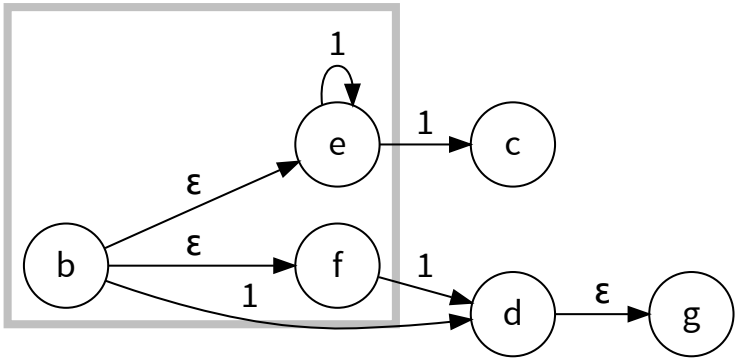

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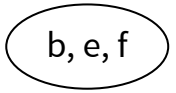

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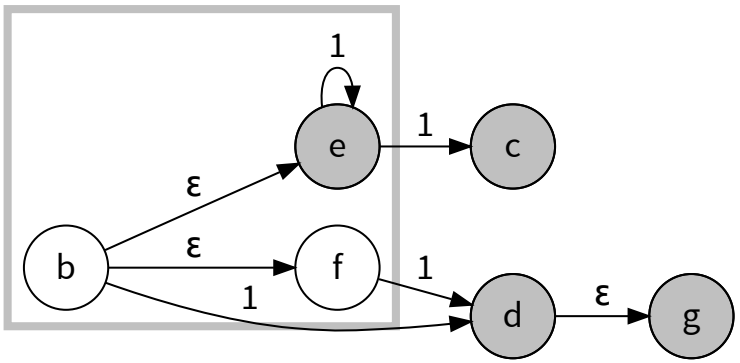

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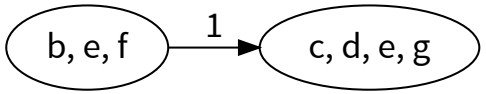
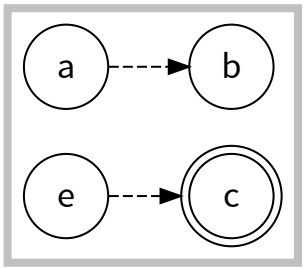


NFAs to DFAs: final states

The final states of the DFA:

Every DFA state that represents a set of NFA states containing a final state.

NFA



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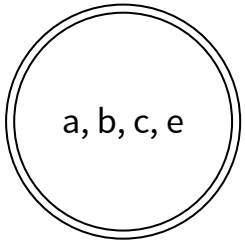

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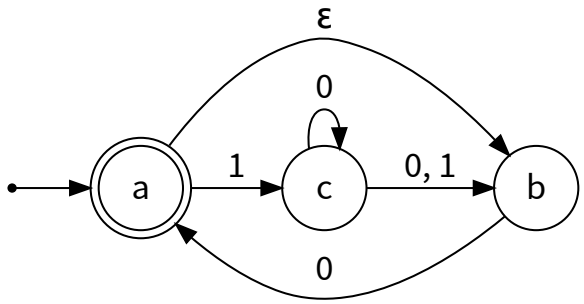

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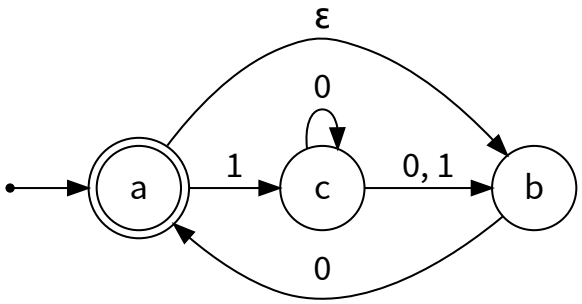

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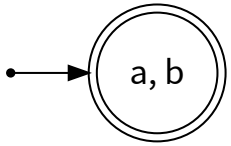

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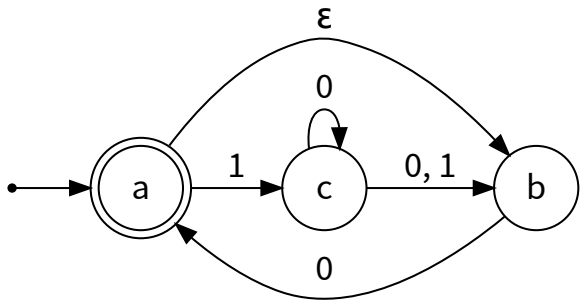

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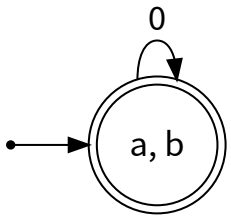

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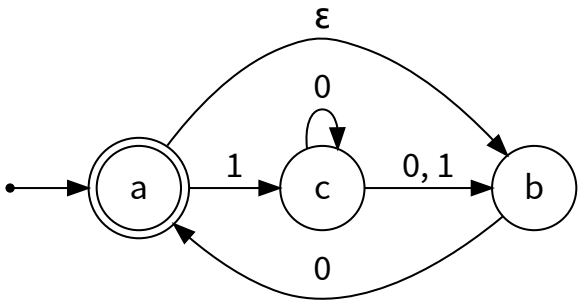

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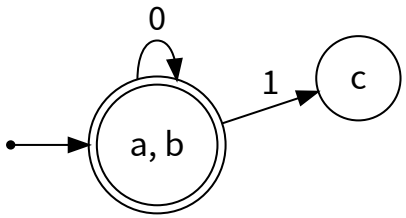

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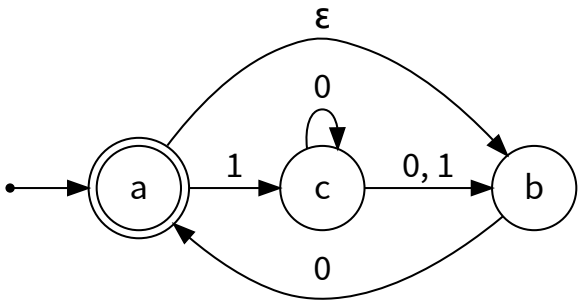

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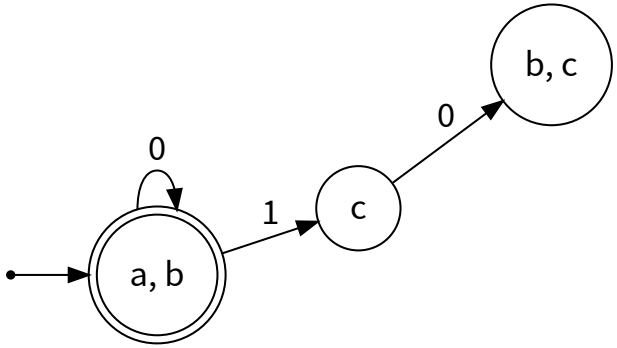

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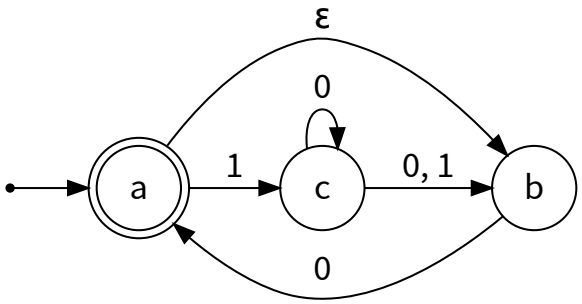

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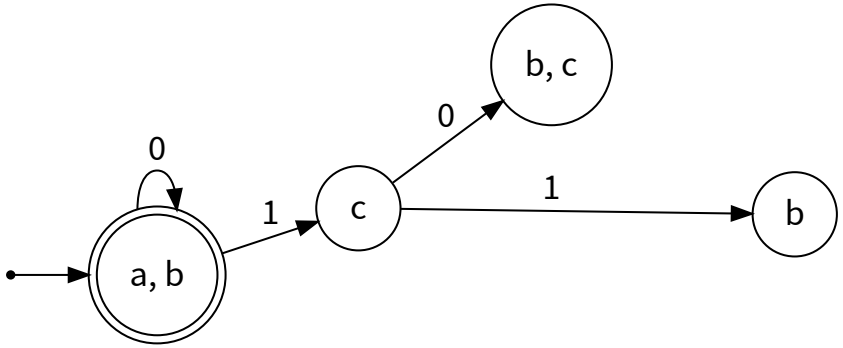

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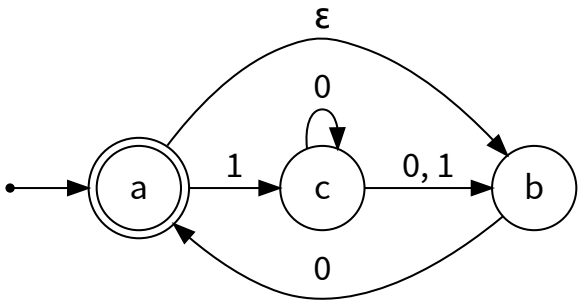

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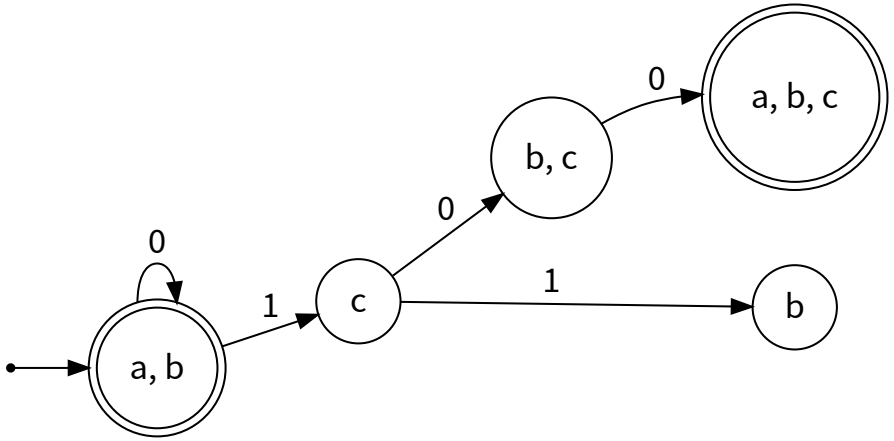

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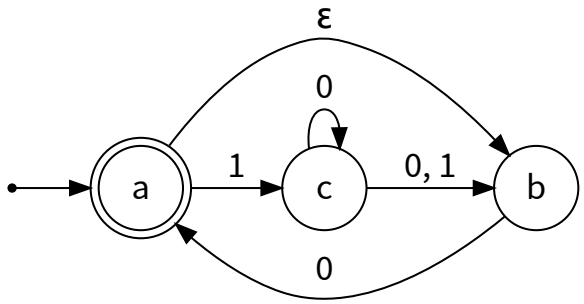

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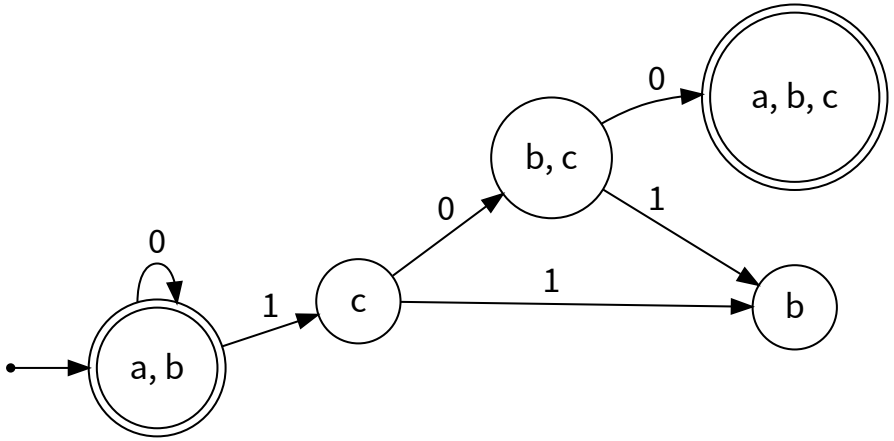

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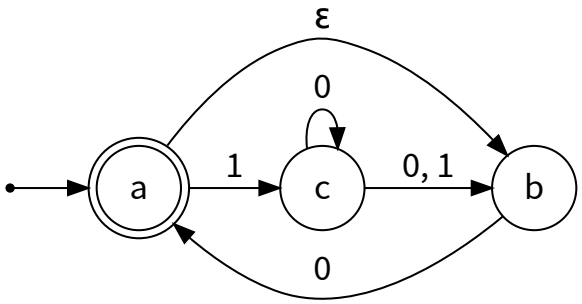

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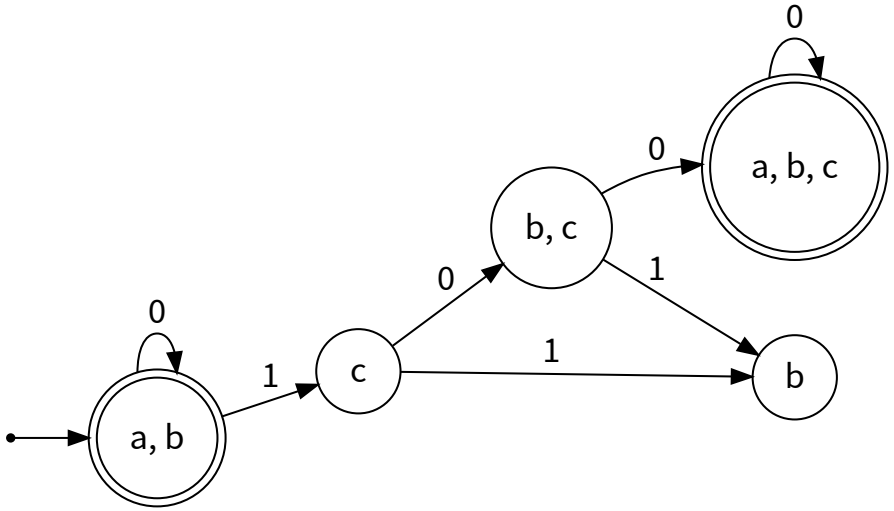

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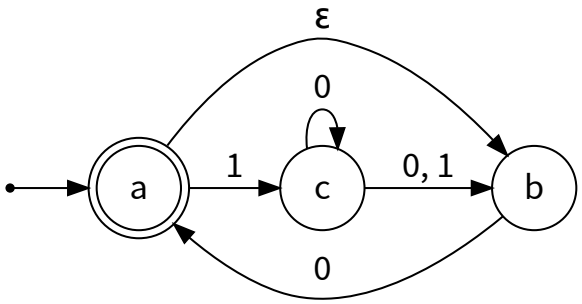

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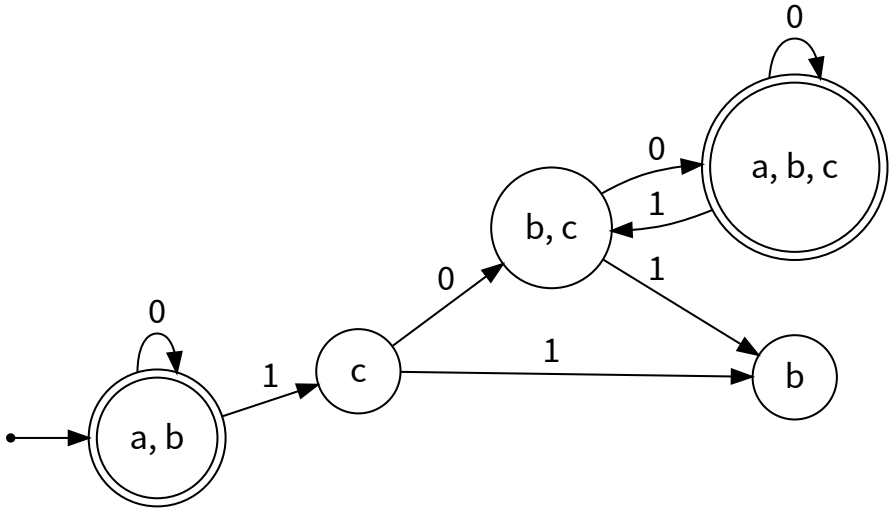

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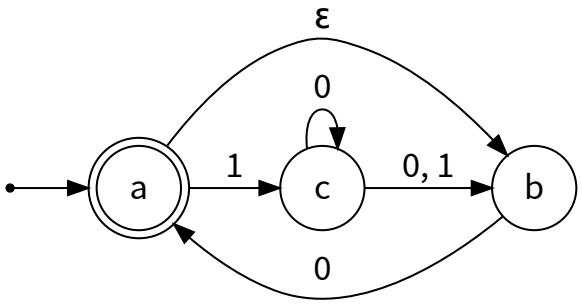

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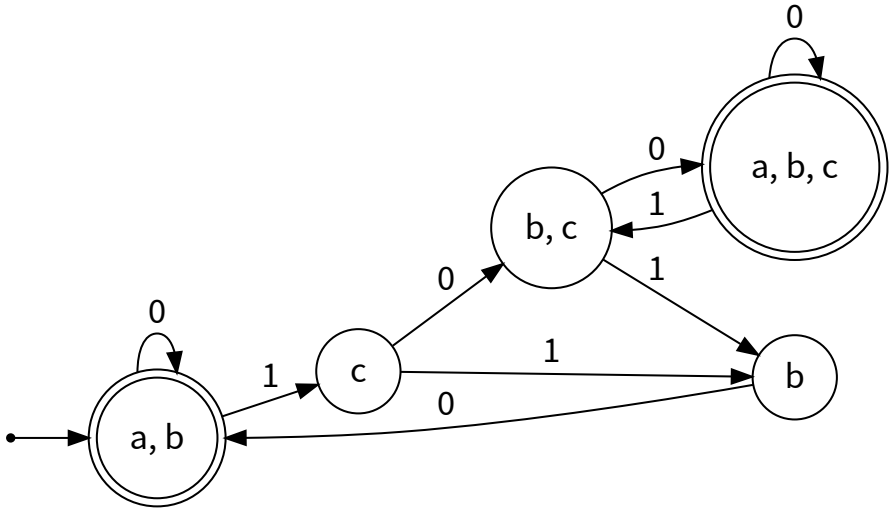

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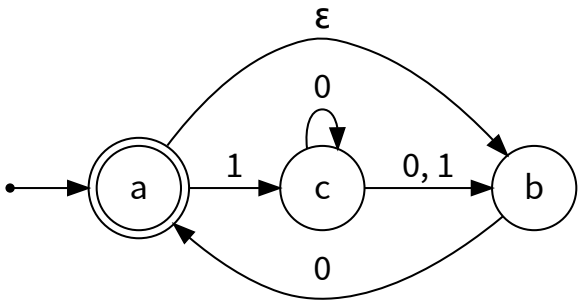

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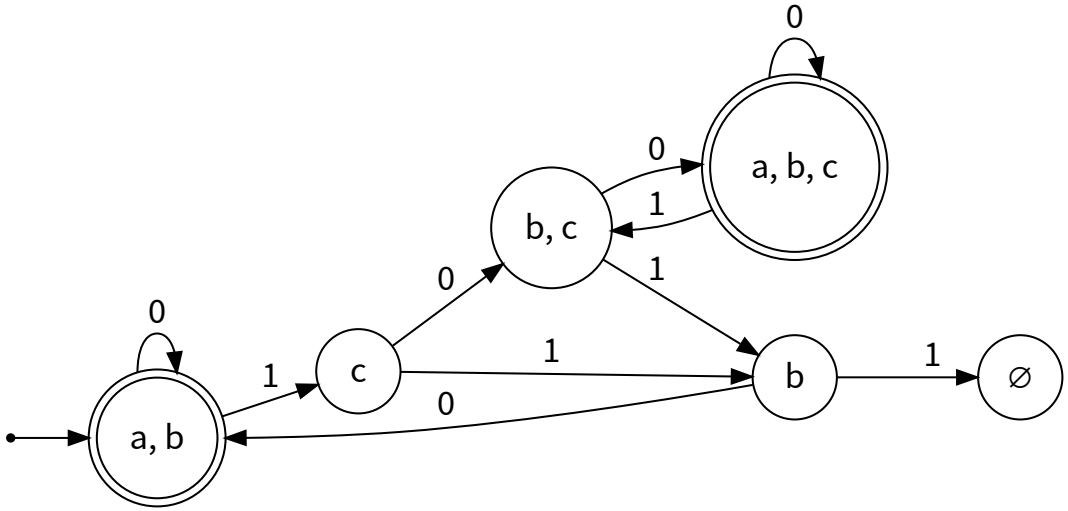

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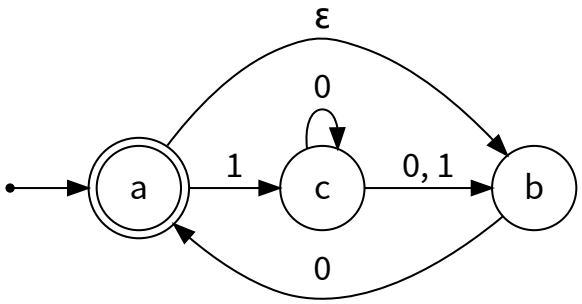

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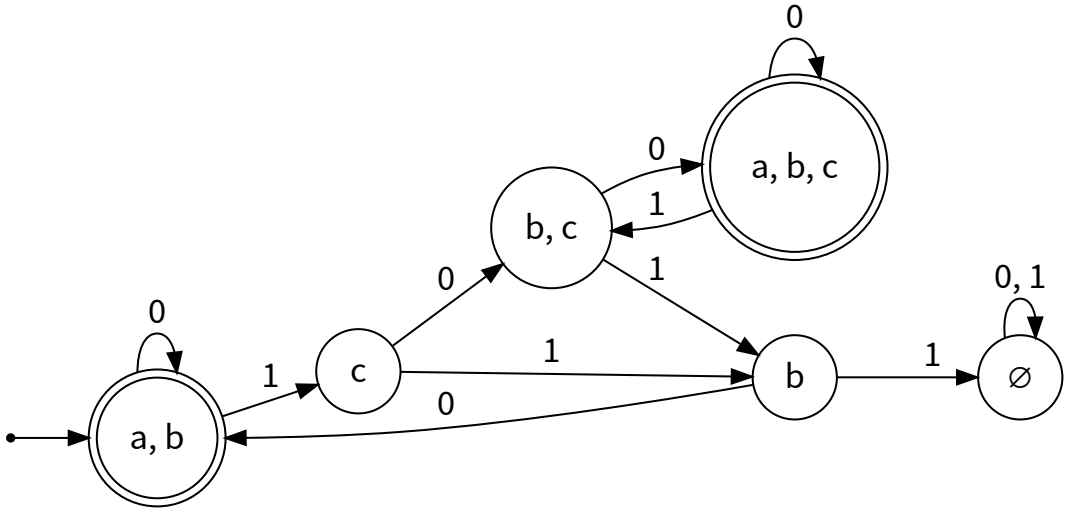

Exponential blow-up in simulating nondeterminism

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Power set of the set of states of the NFA.

n-state NFA yields DFA with up to 2^n states.

We saw an example of this worst case outcome.

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The famous “P=NP?” question asks whether a similar blow-up is always necessary to get rid of nondeterminism for polynomial-time algorithms.

DFAs ≡ NFAs ≡ regular expressions

They are all the same and represent *regular languages*.

Equivalence of DFAs, NFAs, and regular expressions

We have shown how to build an optimal DFA for every regular expression.

Build an NFA.

Convert the NFA to a DFA using the subset construction.

Minimize the resulting DFA.

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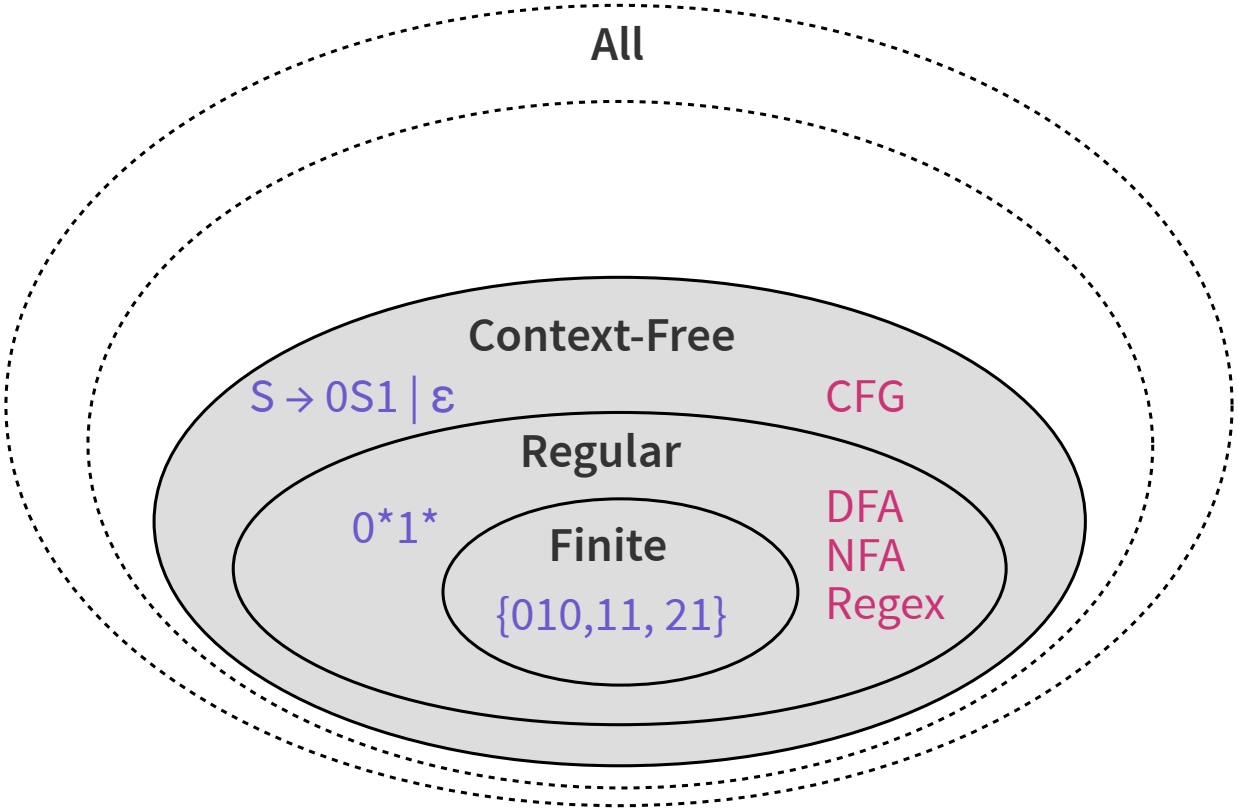
Languages represented by NFAs, DFAs, and regular expressions are called *regular languages*.

Languages and representations

Regular, context-free, and other languages.

A hierarchy of languages and representations



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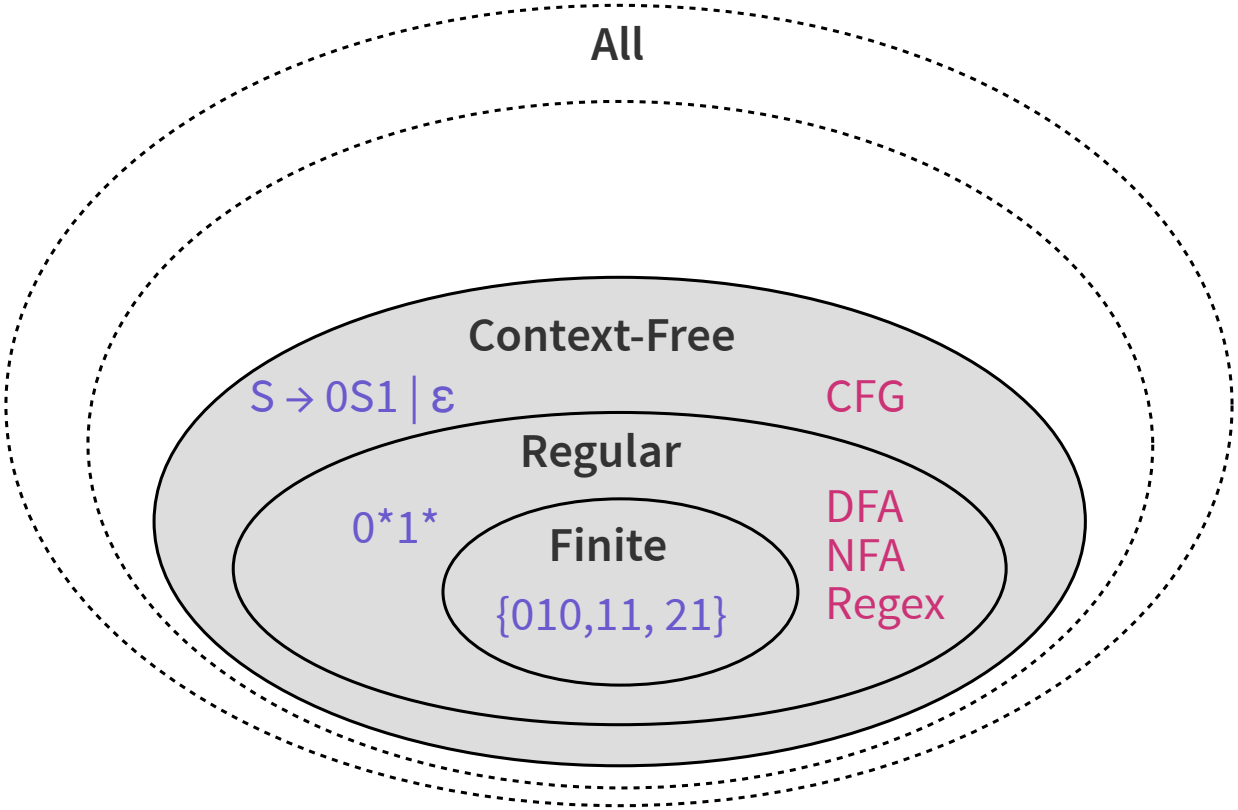

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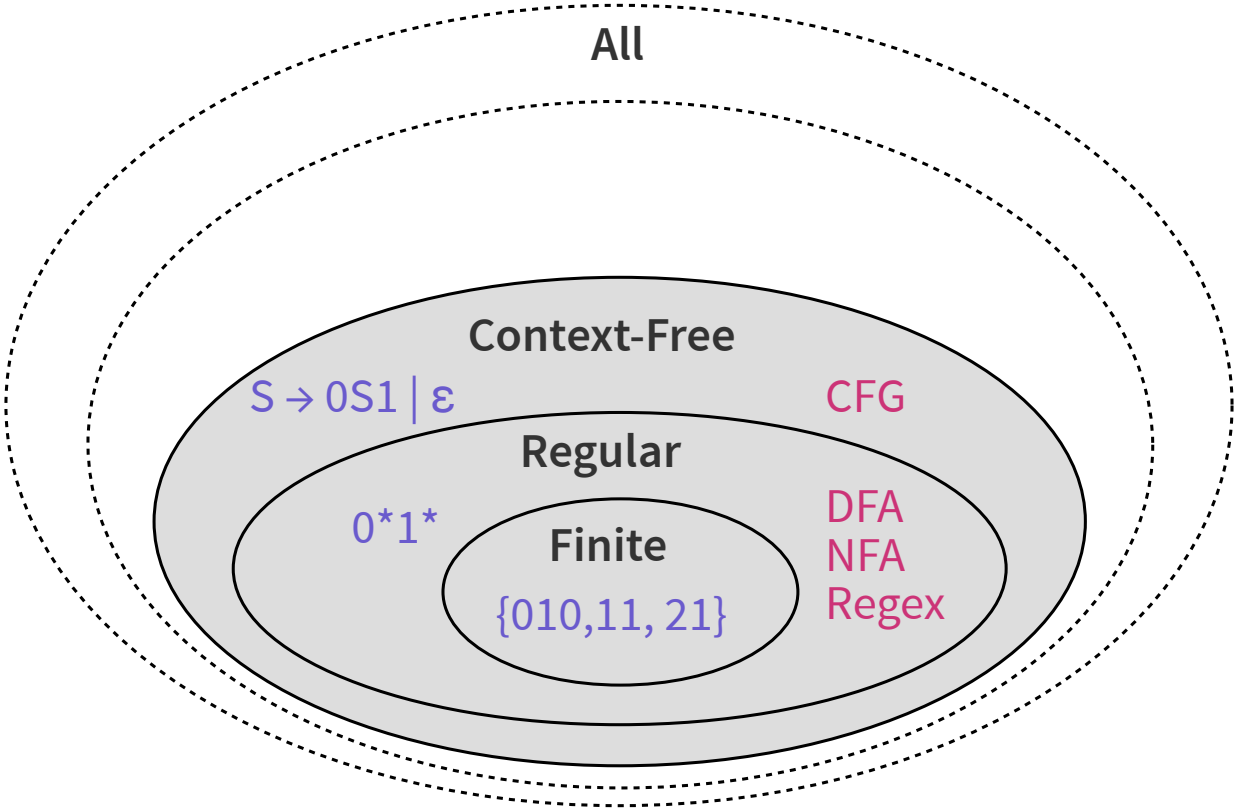

In HW8, you'll (almost :) prove that regular languages are context-free. How do we prove that all finite languages are regular? By showing how to construct a DFA/NFA/RegEx for every finite language!

Converting a finite language to a regular expression

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Convert each string in the language $\mathbf{L}$ to a regular expression.

This is just each string itself.

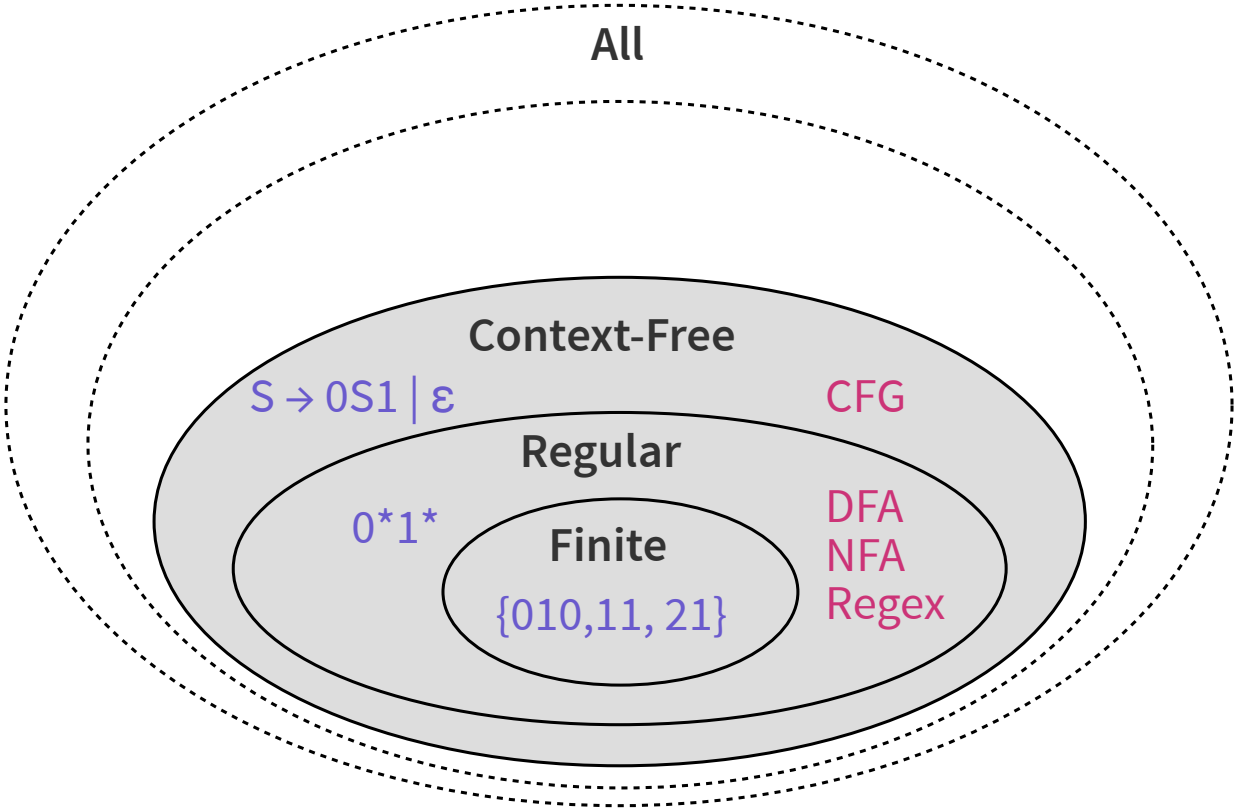
Then put these regular expressions together using the $\cup$ operator.

The resulting regular expression accepts exactly the strings in L.

Example

$$\{010, 11, 21\} \longrightarrow 010 \cup 11 \cup 21$$

Back to languages and representations …



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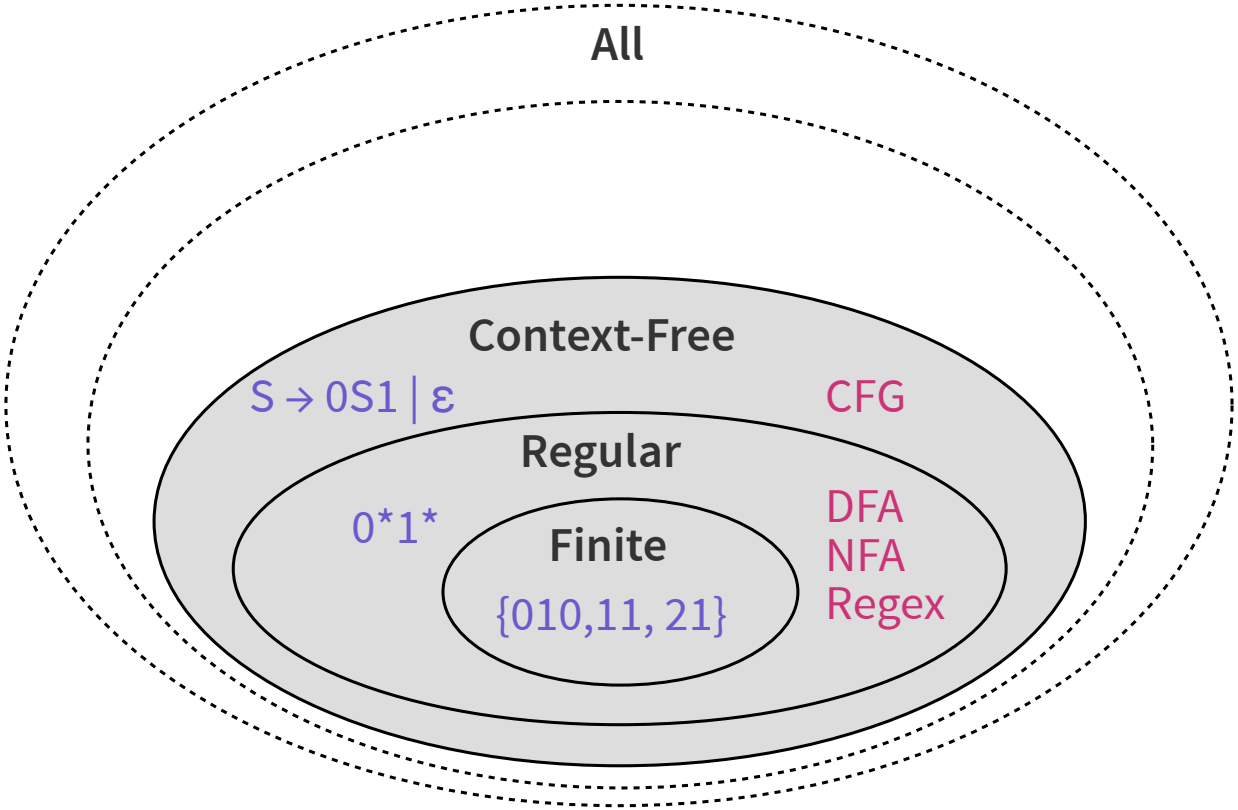

We saw in Lecture 21 that the language B of all binary palindromes can be represented by a CFG. We also said that B can't be represented by any regular expression. How would you prove that?

Why isn't $\mathbf{B}$ (binary palindromes) a regular language?

If $\mathbf{B}$ were regular, we could express it as a DFA/NFA/RegEx.

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This is the intuition for why B is not regular. Let's see how to turn this intuition into a formal proof.

A strategy for proving that $\mathbf{B}$ is not regular

Proof by contradiction:

Assume that B is regular.

Therefore, there is a DFA M that recognizes B.

Show that M accepts or rejects a string it shouldn't.

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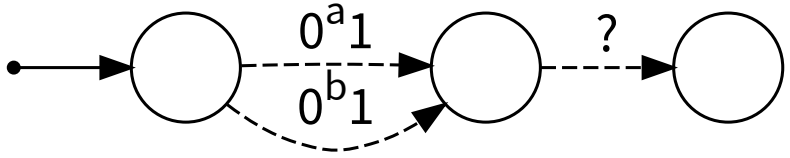

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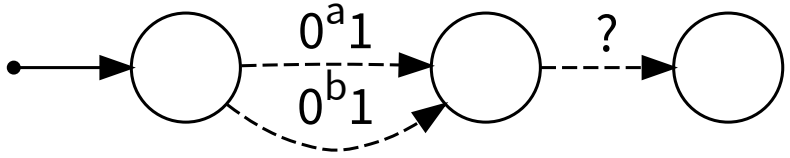

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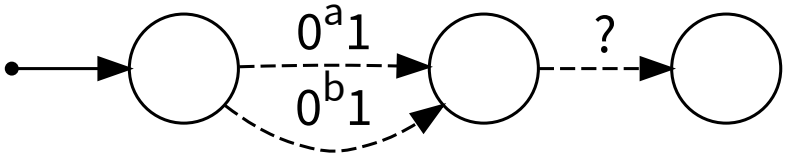

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We choose an infinite set S of prefixes. This choice must ensure that for *every pair* of prefixes in $s_a \neq s_b \in S$, there is a suffix t such that one of of s_at, s_bt is in B but not the other.

$$S = \{1, 01, 001, 0001, \ldots\}$$
$$= \{0^n1 : n \geq 0\}$$

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Important: We don't get to choose a and b! We just know they exist.

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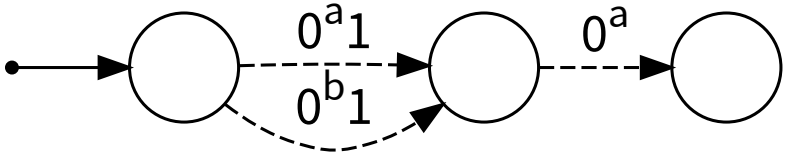

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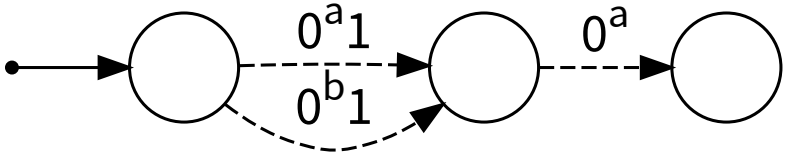

Since $0^a 1$ and $0^b 1$ end in the same state, so do $0^a 10^a$ and $0^b 10^a$, call it q. But then M must make a mistake: q needs to be an accept state since $0^a 10^a \in B$, but then M would accept $0^b 10^a \notin B$, which is an error. □

Proving irregularity

A proof template for showing that a language is not regular.

A template for proving that a language $\mathbf{L}$ is not regular

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Suppose that for a language L, the set S is a *largest* set of prefix strings with the property that for every pair $s_a \neq s_b \in S$, there is some string t such that one of $s_a t$, $s_b t$ is in L but the other isn't.

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If S is finite, then the minimal DFA for L has precisely $|S|$ states, one reached by each member of S.

Summary

DFAs ≡ NFAs ≡ regular expressions.

We've shown how to go from a regular expression to an NFA to a DFA.
(And going from an NFA to a DFA can lead to exponential blow-up.)
But we won't show how to go from an NFA/DFA to a regular expression.

Finite ⊂ regular ⊂ context-free languages.

To show that a language is regular, construct a DFA/NFA/RegEx for it.
To show that it is not regular, use the proof method from this lecture.