CSE 311 Lecture 01: Propositional Logic

Emina Torlak and Kevin Zatloukal

Topics

About CSE 311

What you will learn and why!

Course logistics

A quick summary now; details on the course webpage.

Propositional logic

A language of reasoning

About CSE 311

What you will learn and why!

Learn the calculus of computation

Logic

How do we describe ideas precisely?

Formal proofs

How can we be sure we’re right?

Number theory

How do we keep data secure?

Relations

How do we organize information?

Finite state machines

How do we design hardware and software?

Turing machines

Are there problems computers can’t solve?

And become a better programmer!

By the end of the course, you will have the key technical tools to …

- reason about difficult problems;
- automate difficult problems;
- communicate ideas, methods, and objectives;
- understand fundamental structures for computer science.

Course logistics

A quick summary now; details on the course webpage.

Instructors: cse311-staff@cs

Emina Torlak

Section A: MWF 09:30-10:20 in SIG 134

Office hours: W 10:30-11:30 in CSE 596

Kevin Zatloukal

Section B: MWF 13:30-14:20 in JHN 102

Office hours: F 14:30-15:30 in CSE 212

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Kevin Zatloukal

Section B: MWF 13:30-14:20 in JHN 102

Office hours: F 14:30-15:30 in CSE 212

Afternoon section will be *recorded*

TAs: cse311-staff@cs

Philip Garrison
Kush Gupta
Sarvagya Gupta
Siddharth Iyer Vaidynathan
Weihan (Joy) Ji
Benjamin Lee
Benjamin MacMillan
Aishwarya Nirmal
Zhiheng (Frank) Qin
Aditya Saraf
Oscar Sprumont
Jason Waataja

Textbook, homework, exams, and grading

Optional textbook

Rosen, *Discrete Mathematics and Its Applications*, 6th Edition, McGraw-Hill.

Homework

Due Wednesday at 23:59 online

Write up individually

Exams

Midterm exam on Wed, Nov 07 in class

Final exam on Mon, Dec 10 at 14:30-16:20 and 16:30-18:20

Grading

Homework: 50%

Midterm: 15-20%

Final: 30-35%

Piazza discussion board

opt out of “careers”

About grades

You may find this hard to accept…

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Questions of the form "will I lose points if…"

- are not worth your time (or mine)
- your TAs will decide this anyway
- if you are really worried, then show more work

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Questions of the form "will I lose points if…"

- are not worth your time (or mine)
- your TAs will decide this anyway
- if you are really worried, then show more work

You are **not in competition** with your classmates.

Propositional logic

A language of reasoning

What is logic and why do we need it?

Logic is a language, like English or Java, with its own

- *words* and rules for combining words into *sentences* (**syntax**), and
- a way to assign *meaning* to words and sentences (**semantics**).

So why learn another language when we know English and Java?

Why not use English?

Turn right here.

We saw her duck.

Visiting relatives can be fun.

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Does “right” mean the direction or now?

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Does “duck” mean the animal or crouch down?

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Natural language can be imprecise.

Why not use Java?

The following method determines . . .

```java
public static boolean mystery(int x) {
    for(int r = 2; r < x; r++) {
        for(int q = 2; q < x; q++) {
            if (r*q == x) {
                return false;
            }
        }
    }
    return (x > 1);
}
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Programs can be verbose and take a while to understand.

Logic is both precise and concise!

We need a language of reasoning to

- state sentences more precisely,
- state sentences more concisely, and
- understand sentences more quickly.

Propositions: the basic building blocks of logic

A *proposition* is a statement that is either true or false.

All cats are mammals.

This is a true proposition.

All mammals are cats.

This is a false proposition.

Are these propositions?

$2 + 2 = 5$

$x + 2 = 5$

Who are you?

Pay attention.

Every positive even integer can be written as the sum of two primes.

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This is a proposition that is false.

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Not a proposition because it’s a command rather than a statement.

Every positive even integer can be written as the sum of two primes.

This is a proposition. Nobody knows its truth value, but it’s unique!

Abstracting atomic propositions with variables

Propositional variables represent *atomic propositions* (“words”).

By convention, we use lower-case letters for these variables: $p, q, r, \ldots$

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Variable	Proposition
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The *truth value* of a propositional variable is either

- T for true, or
- F for false.

Making compound propositions with logical connectives

We combine atomic propositions into *compound propositions* ("sentences") using *logical connectives*.

Here is a compound proposition about Garfield:

> *Garfield has black stripes **if** he is an orange cat **and** likes lasagna, **and** he is an orange cat **or** does **not** like lasagna.*

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> *Garfield has black stripes* ***if*** *he is an orange cat* ***and*** *likes lasagna,* ***and*** *he is an orange cat* ***or*** *does* ***not*** *like lasagna.*

Let's see how to express it in logic using our atomic propositions:

p = "Garfield has black stripes."
q = "Garfield is an orange cat."
r = "Garfield likes lasagna."

From English to logic

Garfield has black stripes ***if*** *he is an orange cat* ***and*** *likes lasagna,* ***and*** *he is an orange cat* ***or*** *does* ***not*** *like lasagna.*

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↓ Step 1: abstract

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(p ***if*** *(q* ***and*** *r))* ***and*** *(q* ***or*** *(**not** r))*

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↓ Step 1: abstract

*(p **if** (q **and** r)) **and** (q **or** (**not** r))*

↓ Step 2: replace English connectives with logical connectives

Logical connectives

Connective	Write as	Read as	True when
Negation	$\neg p$	"not p"	p is false
Conjunction	$p \wedge q$	"p and q"	both p and q are true
Disjunction	$p \vee q$	"p or q"	at least one of p, q is true
Exclusive Or	$p \oplus q$	"either p or q"	exactly one of p, q is true
Implication	$p \rightarrow q$	"if p then q"	p is false, or both p, q are true
Biconditional	$p \leftrightarrow q$	"p if and only if q"	p, q have the same truth value

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*(p **if** (q **and** r)) **and** (q **or** (**not** r))*

$((q \wedge r) \rightarrow p) \wedge (q \vee (\neg r))$

Understanding logical connectives with truth tables

Connective	Write as	Read as	True when
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p	$\neg p$
F	
T	

p	q	$p \wedge q$
F	F	
F	T	
T	F	
T	T	

p	q	$p \vee q$
F	F	
F	T	
T	F	
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F	T
T	F

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F	F	
F	T	
T	F	
T	T	

p	q	$p \vee q$
F	F	
F	T	
T	F	
T	T	

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F	F	
F	T	
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F	T
T	F

p	q	$p \wedge q$
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F	T	F
T	F	F
T	T	T

p	q	$p \vee q$
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p	$\neg p$
F	T
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p	q	$p \wedge q$
F	F	F
F	T	F
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p	q	$p \vee q$
F	F	F
F	T	T
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p	q	$p \oplus q$
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p	$\neg p$
F	T
T	F

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	F

Understanding implication with a truth table

Connective	Write as	Read as	True when
Implication	$p \to q$	"if p then q"	p is false, or both p, q are true

p	q	$p \to q$
F	F	
F	T	
T	F	
T	T	

Understanding implication with a truth table

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Implication	$p \to q$	"if p then q"	p is false, or both p, q are true

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

Understanding implication as promises

It's useful to think of implications as promises. That is "Did I lie?"

If it's raining, then I have my umbrella.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

	It's raining	It's not raining
I have my umbrella		
I don't have my umbrella		

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It's useful to think of implications as promises. That is "Did I lie?"

If it's raining, then I have my umbrella.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

	It's raining	It's not raining
I have my umbrella	Truth	Truth
I don't have my umbrella	Lie	Truth

The only lie is when:

- It's raining AND
- I don't have my umbrella

Understanding implication: it's not causal!

Are these true?

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

$2 + 2 = 4 \longrightarrow$ earth is a planet

$2 + 2 = 5 \longrightarrow$ 26 is prime

Understanding implication: it's not causal!

Are these true?

$2 + 2 = 4 \rightarrow$ earth is a planet

The fact that the atomic propositions "2 + 2 = 4" and "earth is a planet" are unrelated doesn't matter! Both are true, so the implication is true as well.

$2 + 2 = 5 \rightarrow$ 26 is prime

p	q	$p \rightarrow q$
F	F	T
F	T	T
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Understanding implication: it's not causal!

Are these true?

$2 + 2 = 4 \rightarrow$ earth is a planet

The fact that the atomic propositions "$2 + 2 = 4$" and "earth is a planet" are unrelated doesn't matter! Both are true, so the implication is true as well.

$2 + 2 = 5 \rightarrow$ 26 is prime

Again, the atomic propositions may or may not be related. Because "$2 + 2 = 5$" is false, the implication is true. Whether 26 is prime or not is irrelevant.

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Understanding implication forward and backward

1. *I have collected all 151 Pokémon if I am a Pokémon master.*
2. *I have collected all 151 Pokémon only if I am a Pokémon master.*

These sentences are implications in opposite directions:

Understanding implication forward and backward

> 1. *I have collected all 151 Pokémon if I am a Pokémon master.*
> 2. *I have collected all 151 Pokémon only if I am a Pokémon master.*

These sentences are implications in opposite directions:

1. Pokémon masters have all 151 Pokémon.
2. People who have 151 Pokémon are Pokémon masters.

So, the implications are:

1. **If** I am a Pokémon master, **then** I have collected all 151 Pokémon.
2. **If** I have collected all 151 Pokémon, **then** I am a Pokémon master.

Understanding implication some more

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

- p implies q
- whenever p is true q must be true
- if p then q
- q if p
- p is sufficient for q
- p only if q
- q is necessary for p

Understanding biconditional (bi-implication)

Connective	Write as	Read as	True when
Biconditional	$p \leftrightarrow q$	"p if and only if q"	p, q have the same truth value

p	q	$p \leftrightarrow q$
F	F	
F	T	
T	F	
T	T	

- p iff q
- p is equivalent to q
- p implies q and q implies p
- p is necessary and sufficient for q

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Biconditional	$p \leftrightarrow q$	"p if and only if q"	p, q have the same truth value

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
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- p iff q
- p is equivalent to q
- p implies q and q implies p
- p is necessary and sufficient for q

Now back to understanding our Garfield sentence …

Garfield has black stripes ***if*** *he is an orange cat* ***and*** *likes lasagna,* ***and*** *he is an orange cat* ***or*** *does* ***not*** *like lasagna.*

p = "Garfield has black stripes."
q = "Garfield is an orange cat."
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↓ Step 1: abstract

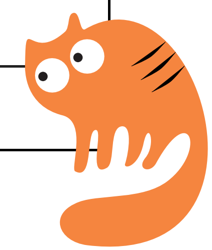
$(p$ **if** $(q$ **and** $r))$ **and** $(q$ **or** $($**not** $r))$

↓ Step 2: replace English connectives with logical connectives

$$((q \land r) \rightarrow p) \land (q \lor (\neg r))$$

Understanding Garfield with a truth table

p	q	r	$\neg r$	$(q \vee (\neg r))$	$(q \wedge r)$	$((q \wedge r) \rightarrow p)$	$((q \wedge r) \rightarrow p) \wedge (q \vee (\neg r))$
F	F	F	T	T	F	T	T
F	F	T	F	F	F	T	F
F	T	F	T	T	F	T	T
F	T	T	F	T	T	F	F
T	F	F	T	T	F	T	T
T	F	T	F	F	F	T	F
T	T	F	T	T	F	T	T
T	T	T	F	T	T	T	T



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Summary

Welcome to CSE 311!

All logistics are on the course webpage.

Propositional logic lets us be concise and precise.

Atomic propositions are "words" in propositional logic.

Compound propositions are "sentences" made with logical connectives.

Implication is tricky: when in doubt, write the truth table!