CSE 311: Foundations of Computing I

Homework 3 (due Wednesday, October 17th at 11:59 PM)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use results from lecture, the theorems handout, and previous homeworks without proof.*

1. All For 1 and One $\forall$ (16 points)

Let the domain of discourse contain only the two object a and b. *For this problem only*, you are allowed to use the following fake equivalence rules

$$\forall x\, P(x) \equiv P(a) \wedge P(b) \qquad \forall \to \wedge$$
$$\exists x\, P(x) \equiv P(a) \vee P(b) \qquad \exists \to \vee$$

(a) [4 Points] Use a chain of equivalences to show that $Q \wedge (\exists x\, P(x)) \equiv \exists x\, Q \wedge P(x)$.

(b) [6 Points] Likewise show that $Q \vee (\exists x\, P(x)) \equiv \exists x\, Q \vee P(x)$.

(c) [2 Points] Are each of these equivalences also true assuming our fake equivalences? Yes or no.

i $Q \wedge (\forall x\, P(x)) \equiv \forall x\, Q \wedge P(x)$

ii $Q \vee (\forall x\, P(x)) \equiv \forall x\, Q \vee P(x)$.

(d) [4 Points] Do the equivalences proven in (a)-(b) hold in every other domain of discourse? Briefly explain why or why not.

2. Opposite Day (18 points)

For each of the following English statements, (i) translate it into predicate logic, (ii) write the negation of that statement in predicate logic with the negation symbols pushed as far in as possible so that any negation symbols are directly in front of a predicate, and then (iii) translate the result of (ii) back to English (natural if possible).

For the logic, let your domain of discourse be cats and activities. You should use only the predicates Loves(x, y) and Likes(x, y) which say that cat x loves or likes (respectively) activity y; the predicates Cat(x) and Activity(x), which say whether x is a cat or activity (respectively); and the predicates $x = y$ and $x \neq y$, which say whether x and y are the same object.

(a) [6 Points] Sleeping is the only activity that Garfield likes.

(b) [6 Points] There is an activity that is loved by some cat but liked by all cats.

(c) [6 Points] Cats that love sleeping don't like some activity.

3. Incontrovertible Spoof (16 points)

Theorem: Given $s \to \neg(p \wedge q)$, $\neg s \to \neg p$, and $(p \wedge q) \vee t$, prove t.
"Spoof":

1.	$s \to \neg(p \wedge q)$	Given
2.	$s \to (\neg p \vee \neg q)$	De Morgan: 1
3.	$\neg s \to \neg p$	Given
4.	$\neg s \to (\neg p \vee \neg q)$	$\vee$ Intro: 3
5.	$(s \to (\neg p \vee \neg q)) \wedge (\neg s \to (\neg p \vee \neg q))$	$\wedge$ Intro: 2, 4
6.	$\neg p \vee \neg q$	Proof by Cases: 5
7.	$\neg(p \wedge q)$	De Morgan: 6
8.	$(p \wedge q) \vee t$	Given
9.	t	$\vee$ Elim: 7, 8

(a) [6 Points] What is the most significant error in this proof? Give the line and briefly explain why it is wrong.

(b) [10 Points] Show that the theorem is true.

4. Quickly Erupting Diatribe (18 points)

(a) [8 Points] Write a formal proof using inference rules of $((p \to q) \wedge (r \to \neg q)) \to (r \to \neg p)$

(b) [10 Points] Write a formal proof using inference rules that, given $r \to \neg(s \wedge p)$, $s \to r$, and $p \oplus q$, the proposition $s \to q$ must also be true. You may use the additional equivalence $a \oplus b \equiv (a \wedge \neg b) \vee (\neg a \wedge b)$, which we will call "Definition of $\oplus$".

5. Quotable Erudite Discussion (12 points)

Using the logical inference rules and equivalences we have given, write a *formal* proof that given $\forall x\ (\exists y\ \neg Q(x, y)) \to R(x)$, $\forall x\ P(x) \to (R(x) \to Q(x, x))$, and $\forall x\ \neg Q(x, x)$, you can conclude that $\exists x\ \neg P(x)$.

6. Quietly Ending Debate (20 points)

Recall that an integer n is even iff there exists an integer a such that $n = 2a$, and it is odd iff there exists an integer a such that $n = 2a + 1$.

(a) [12 Points] Give a formal proof that, if n is odd and m is even, then $n + m$ is odd. In addition to the rules given in class, you can also rewrite algebraic expressions to equivalent ones using the rule "Algebra."

(b) [8 Points] Write your proof from part (a) as an English proof.

7. Extra Credit: Aarh! Me Hearties (0 points)

Five pirates, called Ann, Brenda, Carla, Danielle and Emily, found a treasure of 100 gold coins. On their ship, they decide to split the coins using the following scheme:

- The first pirate in alphabetical order becomes the chief pirate.
- The chief proposes how to share the coins, and all other pirates (excluding the chief) vote for or against it.
- If 50% or more of the pirates vote for it, then the coins will be shared that way.
- Otherwise, the chief will be thrown overboard, and the process is repeated with the pirates that remain.

Thus, in the first round Ann is the chief: if her proposal is rejected, she is thrown overboard and Brenda becomes the chief, etc; if Ann, Brenda, Carla, and Danielle are thrown overboard, then Emily becomes the chief and keeps the entire treasure.

The pirates' first priority is to stay alive: they will act in such a way as to avoid death. If they can stay alive, they want to get as many coins as possible. Finally, they are a blood-thirsty bunch, if a pirate would get the same number of coins if she voted for or against a proposal, she will vote against so that the pirate who proposed the plan will be thrown overboard.

Assuming that all 5 pirates are intelligent (and aware that all the other pirates are just as aware, intelligent, and bloodthirsty), what will happen? Your solution should indicate which pirates die, and how many coins each of the remaining pirates receives.