CSE 311: Foundations of Computing I

Homework 2 (due Wednesday, Oct 10 at 11:59 PM)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use results from lecture, the theorems handout, and previous homeworks without proof.*

1. They're All The Same (24 points)

Prove the following assertions using equivalences.

In general, you should use only a single equivalence per line in your proof, but you can use commutativity and associativity an arbitrary number of times in one line. State the name of each equivalence that you apply.

(a) [6 Points] $p \to (q \to r) \equiv (p \land q) \to r$.

(b) [6 Points] $(p \land q) \to r \equiv q \to (p \to r)$.

(c) [12 Points] $(\neg p \to q) \land (p \to q) \equiv q$. (We will call this fact "Proof by Cases".)

2. Smokey Old Boole Rooms (10 points)

Prove that $(X \cdot Y) + ((X' \cdot Y') + Y)' = X$ using the axioms and theorems of boolean algebra.

As above, you should use only a single axiom or theorem per line in your proof (stating its name), with exceptions for commutativity and associativity.

3. Clear Countin' Mornings (22 points)

In lecture 4, we considered a combinational logic example about days of class.

(a) [6 Points] Write c_0 in the product of sum form.

(b) [10 Points] Simplify the sum of product forms of c_1 using boolean algebra axioms and theorems. Make sure to cite which axioms and theorems you are using when simplifying.

(c) [6 Points] In Lecture 4, we discussed the case when the output of the `classesLeft` function is 3, which occurs exactly when the proposition $c_0 \land c_1$ is true. Write down the propositional formulas, with c_0 and c_1 as the atomic propositions, that are true exactly the output of the `classesLeft` function is

   (a) an integer of the form 2^n (that is, a power of 2),
   (b) an integer of the form $2^n - 1$ (that is, the predecessor of a power of 2),
   (c) an integer greater than 1.

4. Drive Them Old Trucks (16 points)

Let the domain of discourse be all vehicles. Let's define the predicates Bicycle(x), Car(x), and Truck(x) to mean that x is a bicycle, car, or truck, respectively. Define the predicates TwoWheels(x) and FourWheels(x) to mean that x has two or four wheels, respectively, and the predicates TwoSeats(x) and FourSeats(x) mean that x has two or four seats, respectively.

Translate each of the following logical statements into English.

(a) [4 Points] $\forall x\, (\text{Car}(x) \to (\text{FourSeats}(x) \land \text{FourWheels}(x)))$

(b) [4 Points] $\neg\exists x\,(\text{Bicycle}(x) \wedge \neg\,\text{TwoWheels}(x))$

(c) [4 Points] $\exists x\,(\text{Truck}(x) \wedge \text{FourSeats}(x)) \wedge \neg\exists x\,(\text{Bicycle}(x) \wedge \text{FourWheels}(x))$

(d) [4 Points] $\forall x\,(\text{TwoWheels}(x) \rightarrow \text{Bicycle}(x)) \wedge \forall x\,(\text{FourWheels}(x) \rightarrow (\text{Car}(x) \oplus \text{Truck}(x)))$

5. Doctors and Lawyers and Such (16 points)

Let the domain of discourse be all people. Let's define the predicates Lawyer(x) and Client(x) to mean that x is a lawyer or a client of a laywer, respectively. Define the predicate Represents(x, y) to mean that x is a lawyer representing client y. You may also assume the existence of an operator "=" such that $x = y$ is true if and only if x and y are the same person.

Translate each of the following English statements into predicate logic.

(a) [4 Points] Not every person is a lawyer or client.

(b) [4 Points] Fred has one lawyer. (Use the constant "Fred" to refer to Fred in your logical statement.)

(c) [4 Points] Not every client has only one lawyer.

(d) [4 Points] Some client has more than one lawyer.

6. Something To Think About (12 points)

The questions below consider the two propositions

$$\exists x\,(P(x) \wedge Q(x)) \quad \text{and} \quad (\exists x\,P(x)) \wedge (\exists x\,Q(x))$$

where P and Q are predicates.

(a) [6 Points] Give examples of predicates P and Q and a domain of discourse so that the two propositions are **not** equivalent.

(b) [6 Points] Give examples of predicates P and Q and a domain of discourse so that they **are** equivalent.

(c) [0 Points] **Extra credit**: What relationship holds between these two propositions? Explain.

7. Extra credit: Why Do I Have To Choose (0 points)

In this problem, you will design a circuit with a minimal number of gates that takes a pair of four-bit integers $(x_3x_2x_1x_0)_2$ and $(y_3y_2y_1y_0)_2$ and returns a single bit indicating whether $x_3x_2x_1x_0 < y_3y_2y_1y_0$. See the following table for some examples.

$x_3x_2x_1x_0$	$y_3y_2y_1y_0$	$x_3x_2x_1x_0 < y_3y_2y_1y_0$
0101	1011	1
1100	0111	0
1101	1101	0

Design such a circuit using at most 10 AND, OR, and XOR gates. You can use an arbitrary number of NOT gates, and a single gate can have multiple inputs. (Extra credit points start at 10 gates, but if you can use fewer, you will get even more points.)