CSE 311: Foundations of Computing I

Homework 1 (due Wednesday, October 3 at 11:59 PM)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use results from lecture, the theorems handout, and previous homeworks without proof.*

1. Parlez-vous Logique? (24 points)

Translate these English statements into logical language, decomposing each sentence as much as possible into atomic propositions.

(a) [10 Points] Use the same atomic propositions to turn all of these sentences into logic.

- i) If the stack is empty, you can push but not pop,
- ii) If the stack is full, you can pop but not push.
- iii) If the stack is neither full nor empty, you can both push and pop.

(b) [6 Points] Translate the following summary of a recent NPR article to logic: If we can cover 20% of the Sahara desert with solar panels and the rest with wind turbines then the desert will not expand and we can generate four times as much electricity as the entire planet consumes right now.

(c) [8 Points] Use the same atomic propositions to turn both of these sentences into logic.

- i) Your program can use either a `LinkedList` or an `ArrayList` to store a sequence of objects.
- ii) The program will perform well if it uses a `LinkedList` and an iterator to access the list elements, but if the program doesn't use an iterator to access the list elements it will perform poorly unless it uses an `ArrayList`.

2. Mind Your p's and q's (16 points)

Use truth assignments to show that the two propositions in each part are not logically equivalent:

(a) [4 Points] $p \vee q$ vs. $\neg(p \wedge q)$.

(b) [4 Points] $(p \oplus q) \vee (p \oplus r)$ vs. $p \vee q \vee r$.

(c) [4 Points] $(\neg p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$ vs. $(q \rightarrow \neg p) \rightarrow (p \rightarrow \neg q)$.

(d) [4 Points] $p \rightarrow (q \rightarrow (r \rightarrow (s \rightarrow p)))$ vs. $p \rightarrow (q \rightarrow (p \rightarrow (s \rightarrow r)))$.

3. Triple A (20 points)

Let the "A" gate operate as follows. If $p = F$ or $q = F$, then $A(p, q, r) = r$. Otherwise, $A(p, q, r) = \neg r$.

Show how to implement the following gates using only A's. You are allowed to use the constants T and F. You are also allowed to use inputs multiple times.

You must *justify your answers* if they are not obvious.

(a) [4 Points] $\neg p$, using only **one** A gate

(b) [4 Points] $p \oplus q$, using only **one** A gate

(c) [4 Points] $p \wedge q$, using only **one** A gate

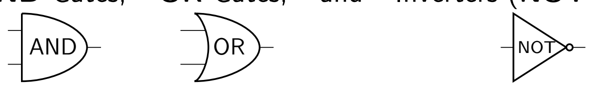
(d) [4 Points] $\neg(p \wedge q)$, using only **one** A gates

(e) [4 Points] $p \vee q$, using at most **three** A gates

4. Portrait of B Gates (10 points)

Using only...

AND Gates, **OR** Gates, and Inverters (**NOT** Gates),



draw the diagram of a circuit with **three inputs** and **two outputs** that computes the function $B(p, q, r)$, defined as follows. If $r = T$, then the outputs are p and q. However, if $r = F$, then the outputs are q and p. In other words, B swaps the two inputs, p and q when $r = F$ and leaves them unchanged otherwise.

5. Majority Rulez (10 points)

Find a compound proposition involving the propositional variables p, q, and r that is true precisely when a majority of p, q, and r are true. Explain why your answer works.

6. The Curious Case of The Lying TAs (10 points)

A new UW CSE student wandered around the Paul Allen building on their first day in the major. They found (as many do) that there is a secret room in its basements. On the door of this secret room is a sign that says:

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All ye who enter, beware!  Every inhabitant of this room is either a TA who always
 lies or a student who always tells the truth!
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(a) [5 Points] The CSE student walked into the room, and two inhabitants walked up to the student. One of them said "at least one of us is a TA." Determine (with justification) all the possibilities for each of the two inhabitants.

(b) [5 Points] Three inhabitants walk up to the CSE student and surround the UW CSE student. One of them says "every TA in this circle has a TA to his immediate right." Determine (with justification) all the possibilities for each of the three inhabitants.

7. EXTRA CREDIT: XNORing (0 points)

For two bits a and b, we define $\texttt{XNOR}(a, b) = \neg(a \oplus b)$. You are given two memory registers, each with the same number of bits. You have an operation, $\texttt{XNOR}(R_1, R_2)$, which takes two registers, R_1 and R_2, performs bitwise `XNOR` between them, and stores the result in R_1.

Show how you can swap the contents of the two registers using a sequence of `XNOR`s without temporary memory registers.