CSE 311: Foundations of Computing I

Section 2: Equivalences and Predicate Logic

0. Equivalences

Prove that each of the following pairs of propositional formulae are equivalent using propositional equivalences.

(a) $p \leftrightarrow q$ $\quad\quad$ $(p \wedge q) \vee (\neg p \wedge \neg q)$

(b) $\neg p \rightarrow (q \rightarrow r)$ $\quad\quad$ $q \rightarrow (p \vee r)$

1. Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent by finding inputs they differ on.

(a) $p \rightarrow q$ $\quad\quad$ $q \rightarrow p$

(b) $p \rightarrow (q \wedge r)$ $\quad\quad$ $(p \rightarrow q) \wedge r$

2. Boolean Algebra

For each of the following parts, write the logical expression using boolean algebra operators. Then, simplify it using axioms and theorems of boolean algebra.

(a) $\neg p \vee (\neg q \vee (p \wedge q))$

(b) $\neg(p \vee (q \wedge p))$

3. Canonical Forms

Consider the boolean functions $F(A, B, C)$ and $G(A, B, C)$ specified by the following truth table:

A	B	C	$F(A,B,C)$	$G(A,B,C)$
1	1	1	1	0
1	1	0	1	1
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1
0	1	0	1	0
0	0	1	0	1
0	0	0	1	0

(a) Write the DNF and CNF expressions for $F(A, B, C)$.

(b) Write the DNF and CNF expressions for $G(A, B, C)$.

4. Translate to Logic

Express each of these system specifications using predicate, quantifiers, and logical connectives.

(a) Every user has access to an electronic mailbox.

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

(c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

(d) At least one router is functioning normally if the throughput is between 100kbps and 500 kbps and the proxy server is not in diagnostic mode.

5. Translate to English

Translate these system specifications into English where $F(p)$ is "Printer p is out of service", $B(p)$ is "Printer p is busy", $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued". Let the domain be all printers and print jobs.

(a) $\exists p\ (F(p) \land B(p)) \rightarrow \exists j\ L(j)$

(b) $(\forall p\ B(p)) \rightarrow (\exists j\ Q(j))$

(c) $\exists j\ (Q(j) \land L(j)) \rightarrow \exists p\ F(p)$

(d) $(\forall p\ B(p) \land \forall j\ Q(j)) \rightarrow \exists j\ L(j)$

6. Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other (or if they are equivalent).

(a) $\forall x\ \forall y\ P(x, y)$ $\qquad$ $\forall y\ \forall x\ P(x, y)$

(b) $\exists x\ \exists y\ P(x, y)$ $\qquad$ $\exists y\ \exists x\ P(x, y)$

(c) $\forall x\ \exists y\ P(x, y)$ $\qquad$ $\forall y\ \exists x\ P(x, y)$

(d) $\forall x\ \exists y\ P(x, y)$ $\qquad$ $\exists x\ \forall y\ P(x, y)$

7. TR∀NSL∀TOR

Express each of these sentences using predicates, quantifiers, and logical connectives. Make sure to define a domain for each part.

(a) There are at least two fluffy dogs in every happy house.

(b) If there a new book or a cheap book by my favorite author in the bookstore, then I will buy it.

(c) All parks have at least one duck pond with more than one duck.