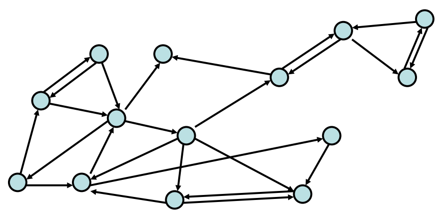
## CSE 311: Foundations of Computing

**Lecture 27a: Relations and Directed Graphs**



## Final Exam

Final Exam Practice is up on the website.
We will have two review sessions:

- Thursday from 4:30 – 7:00 in EEB 105
- Sunday from 1:00 – 4:00 in EEB 105

Enjoy!

## Epsilon Closure?

One of the major reasons that epsilonClosure was so difficult is that we lacked a way of communicating ideas about the “arrows” in an FSM

## Epsilon Closure?

Remember, this course is about the FOUNDATIONS for computing.

We want to give you clean, concise ways of talking about things.

## Epsilon Closure?

Last lecture, we talked about functions as a way of discussing infinity.

Now, let’s generalize functions.

f(x) = y

## Relations

Let A and B be sets,
A binary relation from A to B is a subset of A × B

R : A → B   {(1,1), (2,2), (2,3)}

Let A be a set,
A binary relation on A is a subset of A × A

## Relations You Already Know!

$\geq$ on $\mathbb{N}$

That is: $\{(x,y) : x \geq y \text{ and } x, y \in \mathbb{N}\}$

$<$ on $\mathbb{R}$

That is: $\{(x,y) : x < y \text{ and } x, y \in \mathbb{R}\}$

$=$ on $\Sigma^*$

That is: $\{(x,y) : x = y \text{ and } x, y \in \Sigma^*\}$

$\subseteq$ on P(U) for universe U

That is: $\{(A,B) : A \subseteq B \text{ and } A, B \in P(U)\}$

## Relation Examples

$R_1 = \{(a, 1), (a, 2), (b, 1), (b, 3), (c, 3)\}$

$R_2 = \{(x, y) : x \equiv y \pmod 5\}$

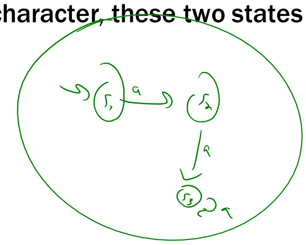
$R_3 = \{(c_1, c_2) : c_1$ is a prerequisite of $c_2\}$

$R_4 = \{(s, c)$ : student s had taken course c $\}$

## Perhaps most importantly…

The "transitions" in a DFA/NFA are a relation!

They say "for a particular character, these two states are "related".



## Properties of Relations

Let R be a relation on A.

R is **reflexive** iff $(a,a) \in R$ for every $a \in A$

R is **symmetric** iff $(a,b) \in R$ implies $(b, a) \in R$

R is **antisymmetric** iff $(a,b) \in R$ and $(b,a) \in R$ implies $a = b$

R is **transitive** iff $(a,b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

## Combining Relations

Let R be a relation from A to B.
Let S be a relation from B to C.

The **composition** of R and S, $S \circ R$ is the relation from A to C defined by:

$S \circ R = \{(a, c) \mid \exists b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$

Intuitively, a pair is in the composition if there is a "connection" from the first to the second.

## Powers of a Relation

Let R be a relation on A.

$R^2 = R \circ R = \{(a, c) : \exists b\, ((a,b) \in R \text{ and } (b, c) \in R\}$

$R^0 = \{(a, c) : a \in A\} = A$

$R^1 = \{(a, b) : (a, b) \in R\} = R$

$R^{n+1} = R^n \circ R$

## Epsilon Closure…

The epsilonClosure of the epsilon transitions is $R^*$

We keep on composing the relation over and over until there's nothing left to add.

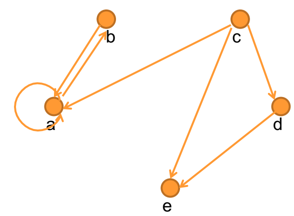
This is called the "transitive closure" of a relation.


$R \cup R^2 \cup R^3 \cup \ldots$

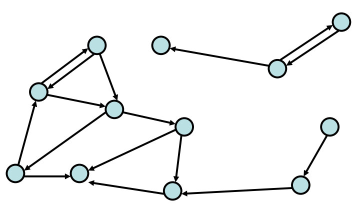
## Representation of Relations

**Directed Graph Representation (Digraph)**

{(a, b), (a, a), (b, a), (c, a), (c, d), (c, e) (d, e) }




## Transitive-Reflexive Closure



Add the minimum possible number of edges to make the relation transitive and reflexive.

The transitive-reflexive closure of a relation R is the connectivity relation R*