CSE 311

Foundations of Computing I

* All slides are a combined effort between previous instructors of the course

DFAs ≡ Regular expressions

We have shown how to build an optimal DFA for every regular expression

- Build NFA
- Convert NFA to DFA using subset construction
- Minimize resulting DFA

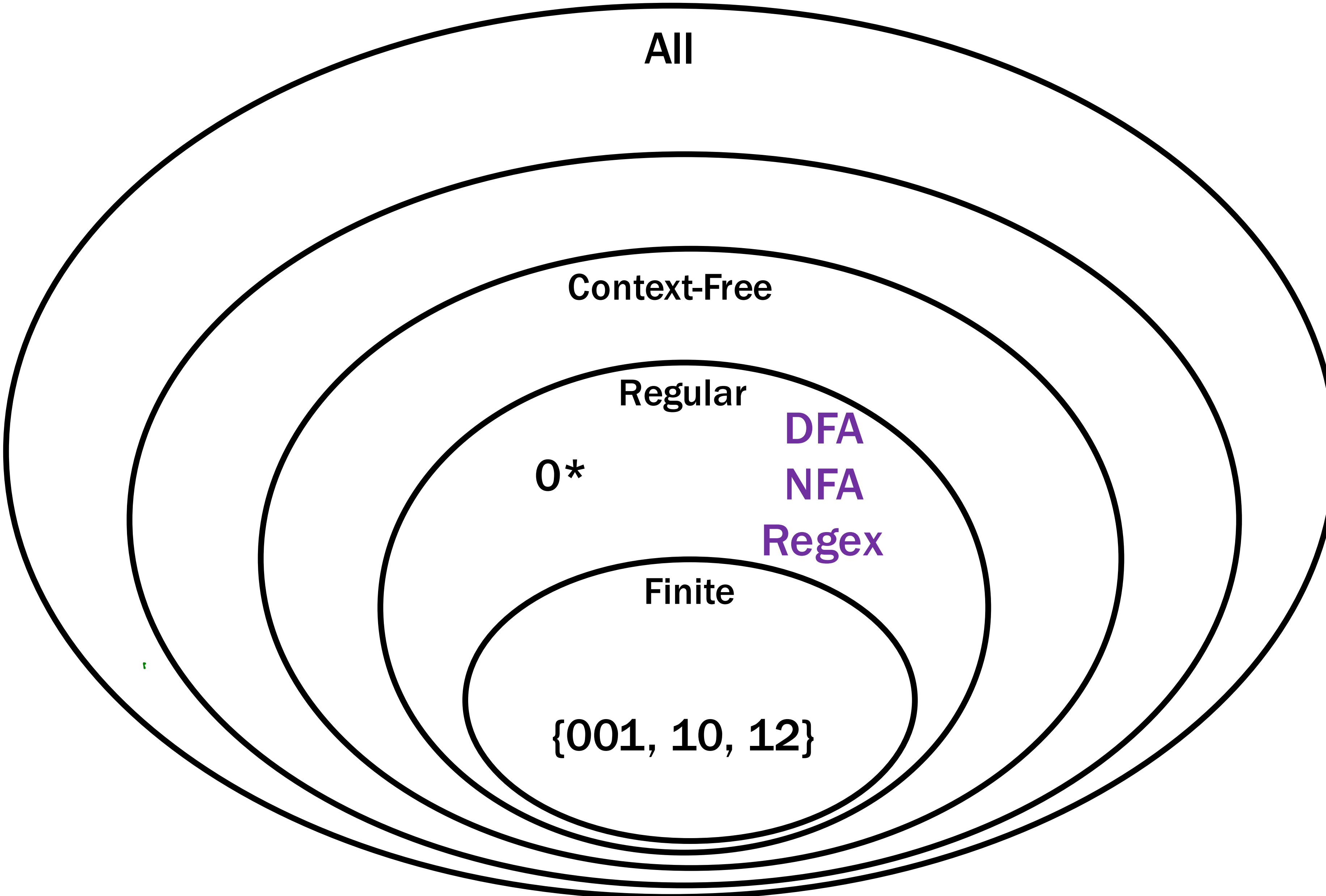
Theorem: A language is recognized by a DFA if and only if it has a regular expression

The second direction will be completely untested. I'm happy to discuss it with you at office hours, but we have more important things to discuss today.

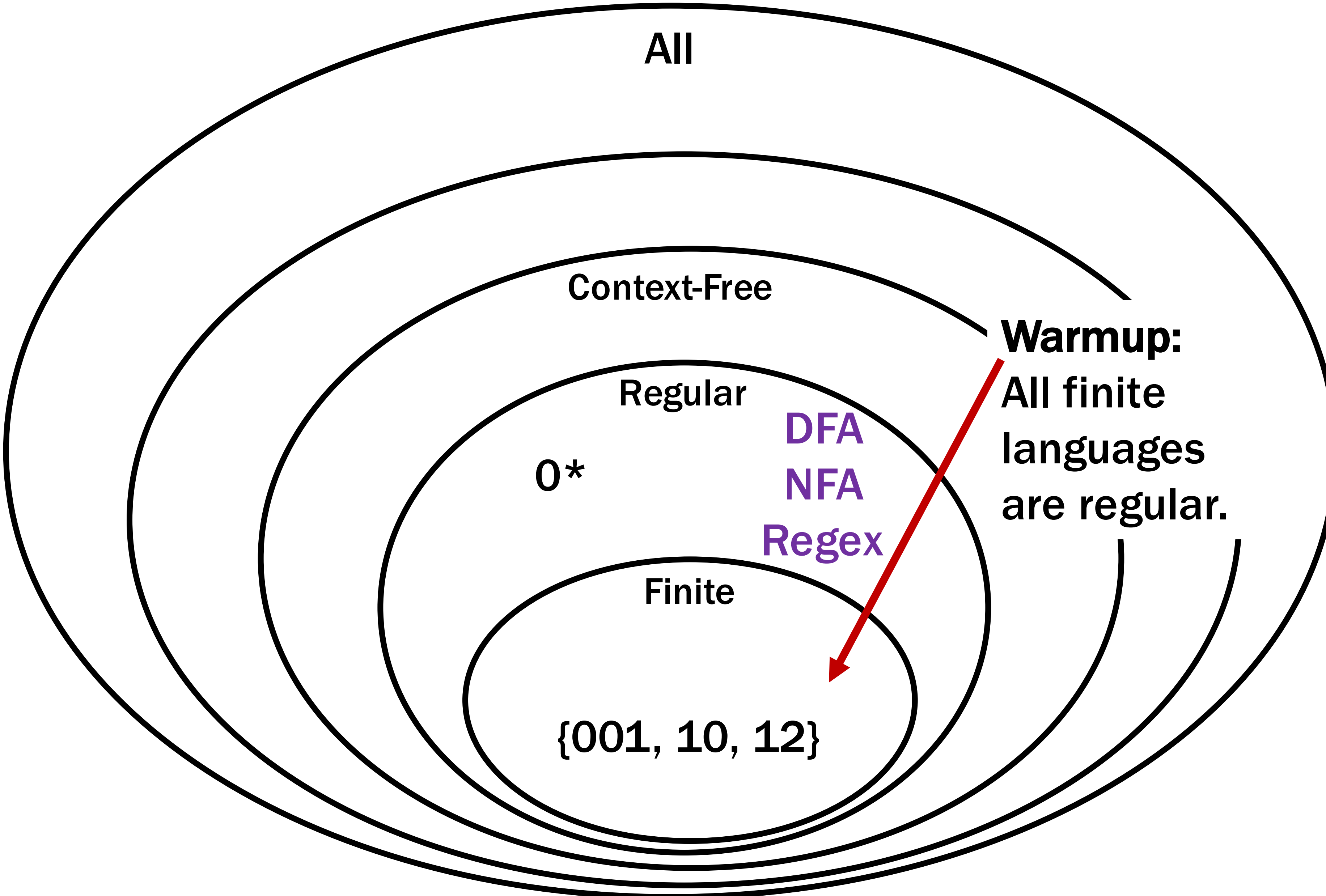
CSE 311: Foundations of Computing

Lecture 25: Limits of FSMs

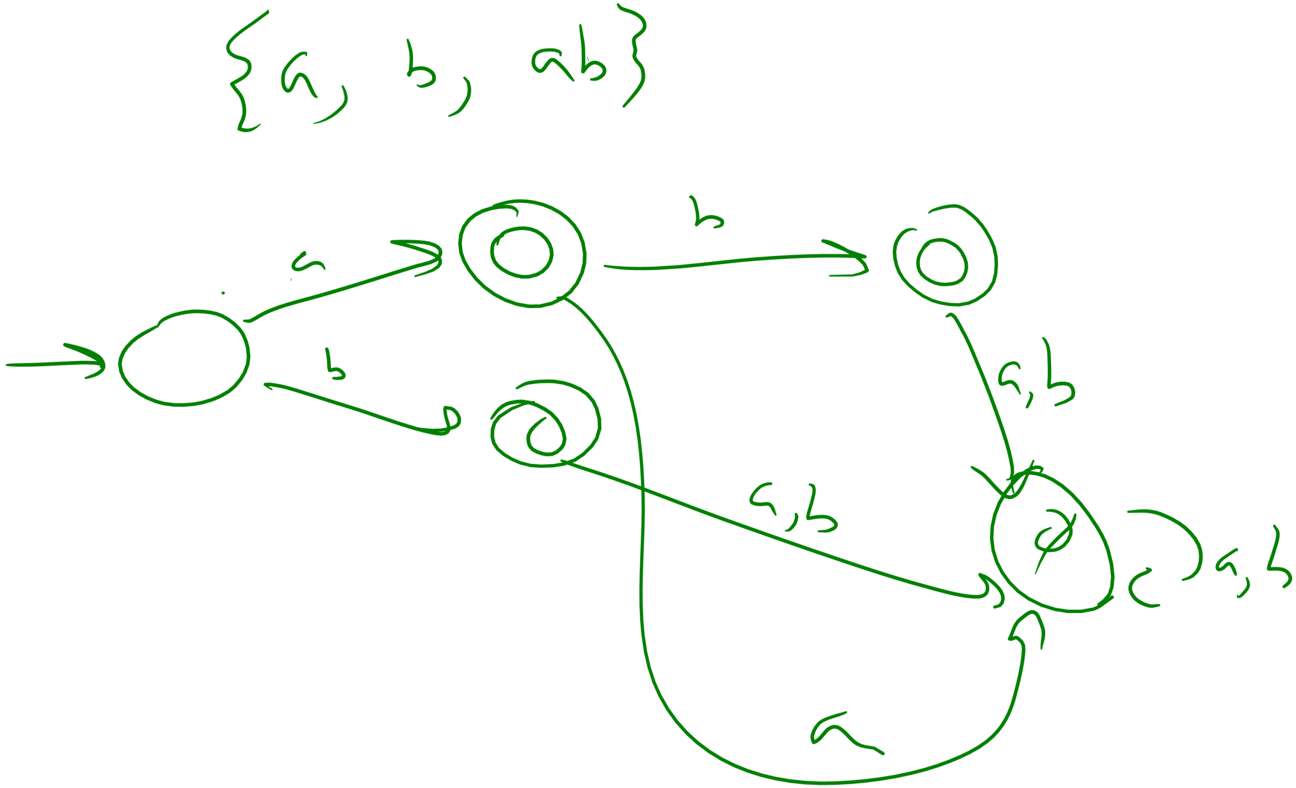
Languages and Machines!



Languages and Machines!



DFAs Recognize Any Finite Language



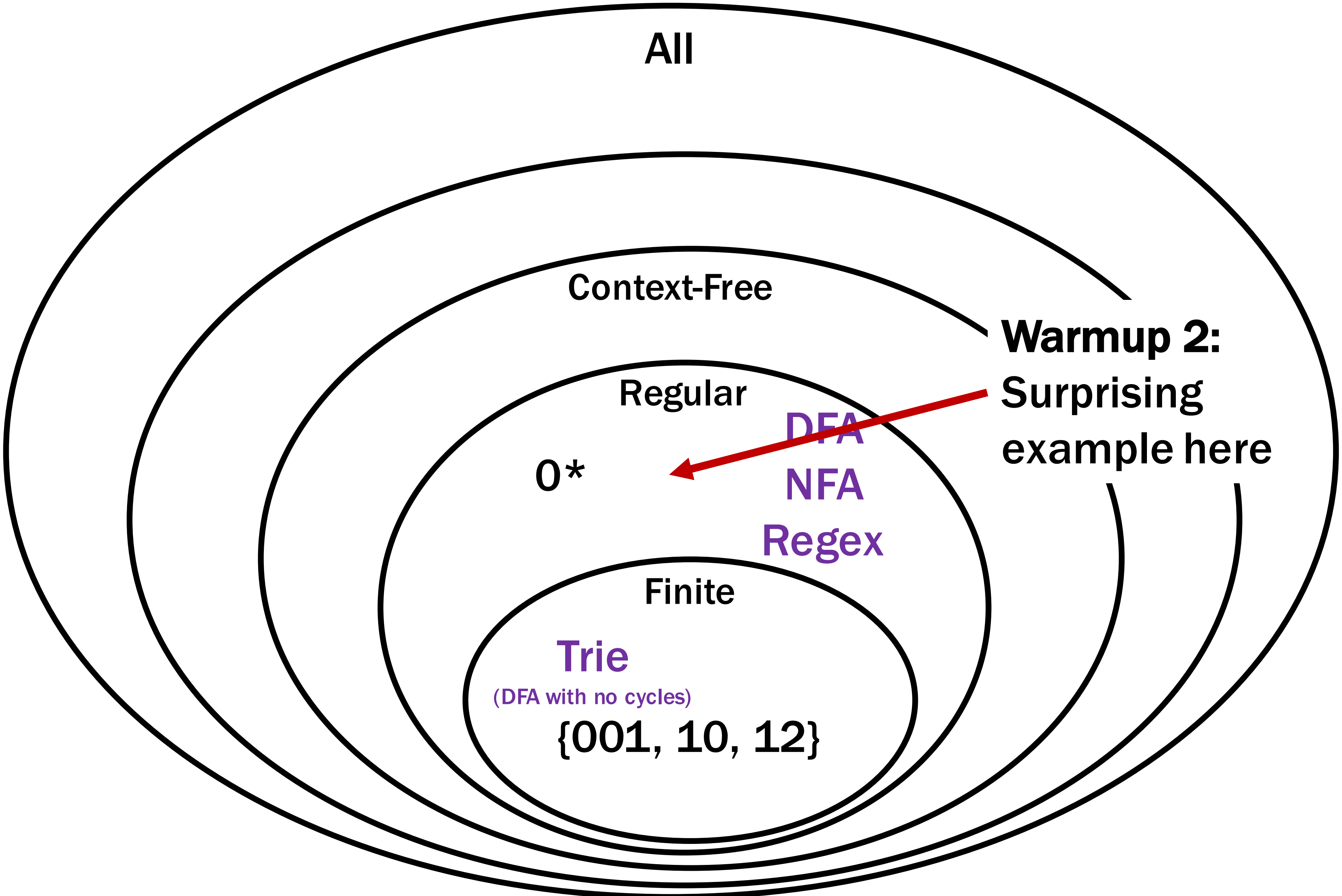
DFAs Recognize Any Finite Language

Construct DFAs for each string in the language.

Then, put them together using the union construction.

This is basically the idea behind a "trie" which is the first data structure you'll implement in 332.

Languages and Machines!



An Interesting Infinite Regular Language

L = {x∈ {0, 1}*: x has an equal number of substrings 01 and 10}.

L is infinite.

0, 00, 000, ...

L is regular.

0110

010110

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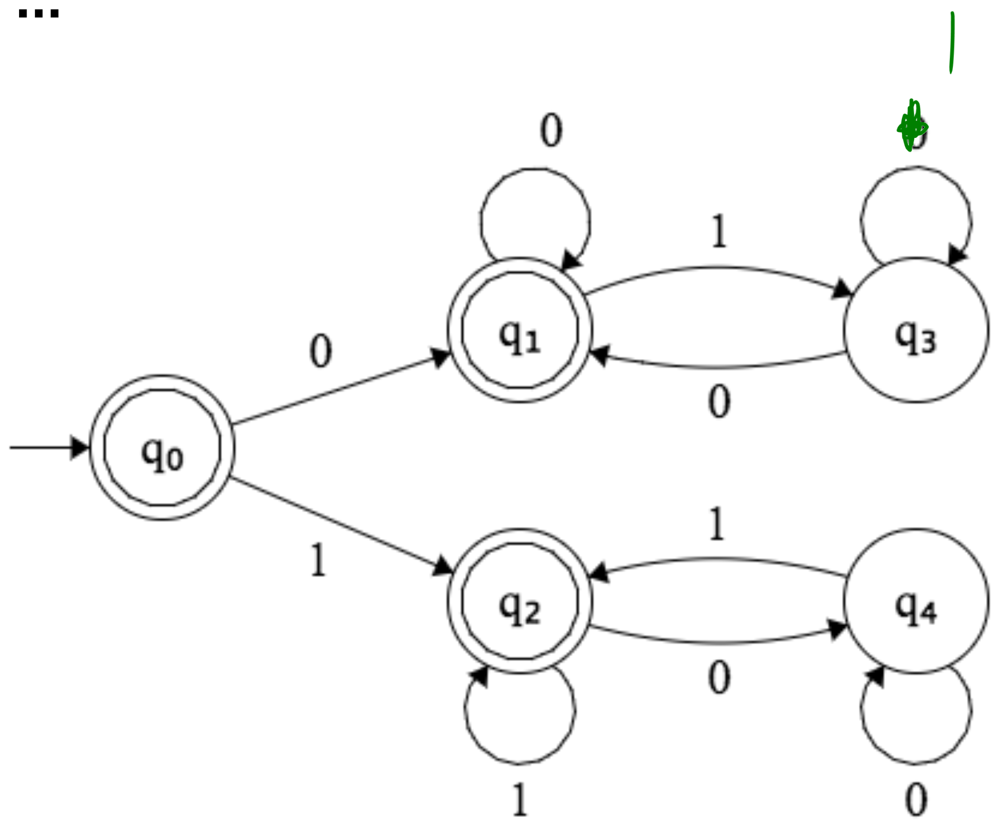


The language of “Binary Palindromes” is Context-Free

$$S \rightarrow \varepsilon \mid 0 \mid 1 \mid 0S0 \mid 1S1$$

We good?

The language of “Binary Palindromes” is Regular

The language of “Binary Palindromes” is Regular

Is it though?

1, 11, 111, 1111 111...0...111

101, 1001

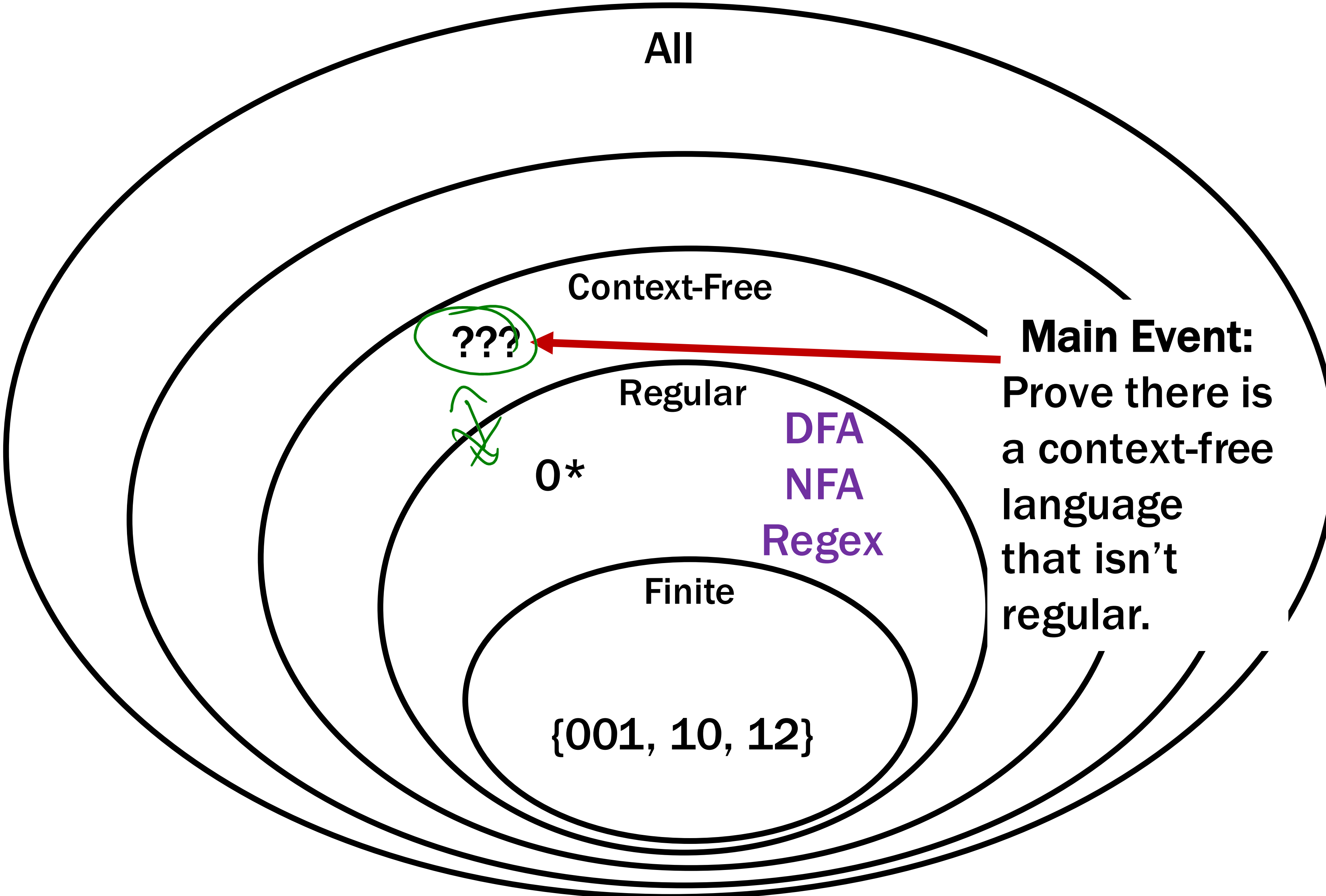
Intuition (NOT A PROOF!):

Q: What would a DFA need to keep track of to decide the language?

A: It would need to keep track of the “first part” of the input in order to check the second part against it

…but there are an infinite # of possible first parts and we only have finitely many states.

Languages and Machines!



Main Event:
Prove there is a context-free language that isn't regular.

B = {binary palindromes} can't be recognized by any DFA

The general proof strategy is:

- Assume (for contradiction) that it's possible.
- Therefore, some DFA (call it M) exists that accepts B

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How can a DFA be "wrong" or "broken"?

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How can a DFA be "wrong" or "broken"?

Just like the errors you were getting on the homework, a DFA is "broken" when it accepts or rejects a string it shouldn't.

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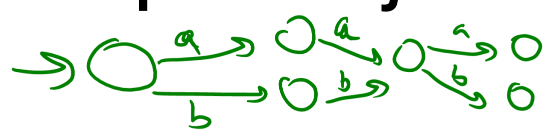
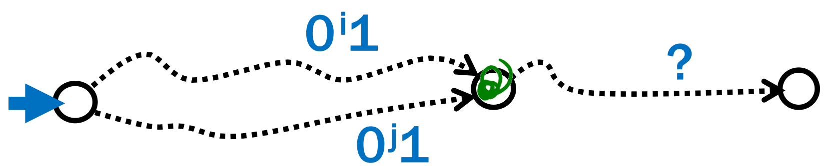


Key Idea 1: If two strings "collide" at any point, an FSM can no longer distinguish between them!

0^i1 and 0^j1 end in the same state

$\rightarrow$ 0^i1x and 0^j1x end in the same state $\forall x \in \Sigma^*$



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Key Idea 1: If two strings "collide" at any point, an FSM can no longer distinguish between them!

Key Idea 2: Our machine has a finite number of states which means if we have infinitely many strings, two of them must collide!

B = {binary palindromes} can't be recognized by any DFA

The general proof strategy is:

- Assume (for contradiction) that it's possible.
- Therefore, some DFA (call it M) exists that accepts B
- We want to show M accepts or rejects a string it shouldn't.
- We choose an **INFINITE** set of "half strings" (which we intend to complete later). It is imperative that if we choose a completion, it "correctly" completes exactly one string.

0 1000	1______	1______	0______
00 1000	10______	01______	01______
000 1000	100______	001______	101______
0000 1000	1000______	0001______	0101______
00000 1000	10000______	00001______	10101______

B = {binary palindromes} can't be recognized by any DFA

0^n...	0...	00...	000...	0000...	00000...
Working:	ε/0/00/000/ 10/110/...	ε/0/00/000/ 100/1100/...	ε/0/00/000/ 1000/11000/...	ε/0/00/000/ 10000/110000/...	ε/0/00/000/ 100000/1100000/...
...0^{n-1}		No! "0" appears in other columns. 0	00	000	0000
...0^n1		Nothing fits here! We can't make this work.			
...10^n	10	Only column with 100! 100	1000	10000	100000

0^n...0^{n-1}	0...	00...	000...
...ε	0	00	
...0			
...00			

Too much

This is already a problem. Since ε works for two different start strings, this is not a valid completion choice.

0^n...0^n1	0...	00...	000...
...01	001	0001	00001
...001			
...0001			

Too little

This is a problem. Since 01 NEVER results in an accept, for any first string, this isn't going to work.

0^n...10^n	0...	00...	000...
...10	010	0010	00010
...100	0100	00100	000100
...1000	01000	001000	0001000

Just right!

There's exactly one green in each column and each row! Perfect!

B = {binary palindromes} can't be recognized by any DFA

Suppose for contradiction that some DFA, M, accepts B.

We show M accepts or rejects a string it shouldn't.

Consider S = {1, 01, 001, 0001, 00001, ...} = $\{0^n 1 : n \geq 0\}$.

Key Idea 2: Our machine has a finite number of states which means if we have infinitely many strings, two of them must collide!

B = {binary palindromes} can't be recognized by any DFA

Suppose for contradiction that some DFA, M, accepts B.

We show M accepts or rejects a string it shouldn't.

Consider S = {1, 01, 001, 0001, 00001, …} = $\{0^n1 : n \geq 0\}$.

Since there are finitely many states and infinitely many strings in S, there exists strings $0^a1 \in S$ and $0^b1 \in S$ that end in the same state.

SUPER IMPORTANT POINT: You do not get to choose what a and be are. Remember, we've proven they exist…we have to take the ones we're given!

	0^a1…	0^b1…
…		

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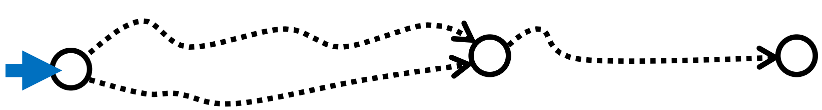
Since there are finitely many states and infinitely many strings in S, there exists strings $0^a1 \in S$ and $0^b1 \in S$ that end in the same state.

Now, consider appending 0^a to both strings.

	$0^a1...$	$0^b1...$
$...0^a$	0^a10^a	0^b10^a

Key Idea 1: If two strings "collide" at any point, an FSM can no longer distinguish between them!



B = {binary palindromes} can't be recognized by any DFA

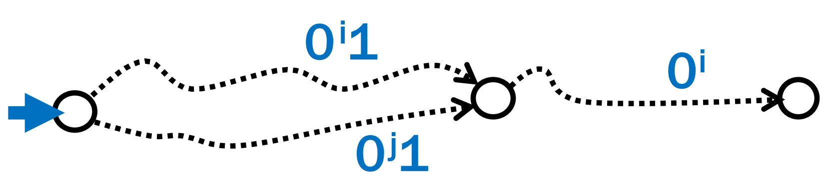
Suppose for contradiction that some DFA, M, accepts B.

We show M accepts or rejects a string it shouldn't.

Consider $S = \{0^n1 : n \geq 0\}$.

Since there are finitely many states and infinitely many strings in S, there exists strings $0^a1 \in S$ and $0^b1 \in S$ that end in the same state with $a \neq b$.

Now, consider appending 0^a to both strings. *Then, since 0^a1 and 0^b1 are in the same state, 0^a10^a and 0^b10^a also end in the same state. Since $0^a10^a \in B$, this state must be an accept state. But, then M accepts $0^b10^a \notin B$.*



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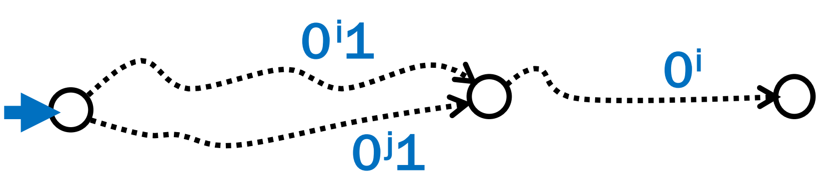

This is a contradiction, because we assumed M accepts B. Since M was arbitrary, **there is no DFA that accepts B.**

Showing a Language L is not regular

1. "Suppose for contradiction that some DFA M accepts L."
2. Consider an **INFINITE** set of "half strings" (which we intend to complete later). It is imperative that every string in our set have a **DIFFERENT, SINGLE** "accept" completion.
3. "Since **S** is infinite and **M** has finitely many states, there must be two strings s_i and s_j in **S** for some $i \neq j$ that end up at the same state of **M**."
4. Consider appending the (correct) completion to one of the two strings.
5. "Since s_i and s_j both end up at the same state of **M**, and we appended the same string **t**, both $s_i t$ and $s_j t$ end at the same state of **M**. Since $s_i t \in L$ and $s_j t \notin L$, **M** does not recognize **L**."
6. "Since **M** was arbitrary, no DFA recognizes **L**."

Prove $A = \{0^n1^n : n \geq 0\}$ is not regular

Suppose for contradiction that some DFA, M, accepts A.

Let S =

Prove $A = \{0^n1^n : n \geq 0\}$ is not regular

Suppose for contradiction that some DFA, M, accepts A.

Let $S = \{0^n : n \geq 0\}$. Since S is infinite and M has finitely many states, there must be two strings, 0^i and 0^j (for some $i \neq j$) that end in the same state in M.

Consider appending 1^i to both strings. Note that $0^i1^i \in A$, but $0^j1^i \notin A$ since $i \neq j$. But they both end up in the same state of M. Since that state can't be both an accept and reject state, M does not recognize A.

Since M was arbitrary, no DFA recognizes A.

Another Irregular Language Example

L = {x ∈ $\{0,1,2\}^*$: x has an equal number of substrings 01 and 10}.

Intuition: Need to remember difference in # of **01** or **10** substrings seen, but only hard to do if these are separated by **2**'s.

Suppose for contradiction that some DFA, M, accepts L.

Let S = {ε, 012, 012012, 012012012, …} = {$(012)^n$: n ∈ ℕ}

Another Irregular Language Example

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Suppose for contradiction that some DFA, M, accepts L.

Let S = {ε, 012, 012012, 012012012, ...} = $\{(012)^n : n \in \mathbb{N}\}$

Since S is infinite and M is finite, there must be two strings $(012)^i$ and $(012)^j$ for some $i \neq j$ that end up at the same state of M. Consider appending string t = $(102)^i$ to each of these strings.

Then, $(012)^i (102)^i \in L$ but $(012)^j (102)^i \notin L$ since $i \neq j$.

So $(012)^i (102)^i$ and $(012)^j (102)^i$ end up at the same state of M since $(012)^i$ and $(012)^j$ do. Since $(012)^i (102)^i \in L$ and $(012)^j (102)^i \notin L$, M does not recognize L.

Since M was arbitrary, no DFA recognizes L.