CSE 311: Sample Homework

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Section AB
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1. Celebration?

Let x be the number of slices of the cake. The problem tells us that we have $x = \frac{x}{4} + \frac{x}{2} + \frac{x}{6} + 3$. Simplifying, we get

$$\begin{aligned} x &= \frac{x}{4} + \frac{x}{2} + \frac{x}{6} + 3 \\ 12x &= 3x + 6x + 2x + 36 \\ x &= 36 \end{aligned}$$

Therefore, we know the cake originally had 36 slices.

2. Propositional Logic

(a) We define our variables as:

- p: I know LaTeX
- q: I can write fancy papers
- r: I can write homework assignments

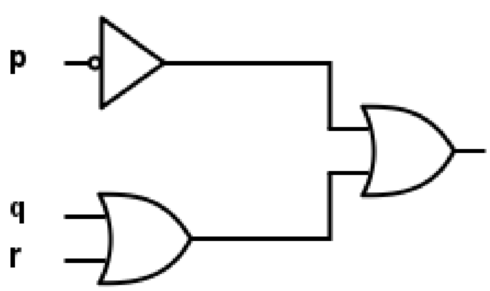
Then, the sentence "If I know LaTeX, then I can write fancy papers, homework assignments, or both." can be expressed as:

$$p \to (q \vee r)$$

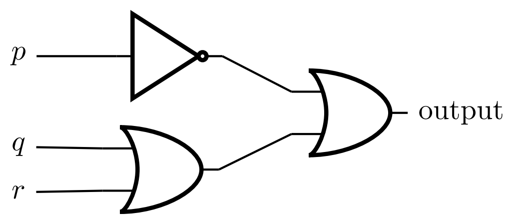
(b) The truth table:

p	q	r	$(q \vee r)$	$p \to (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

(c) The circuit (using an embedded image):



The circuit (using the Circuitikz package):



(d) The code is as follows:

```java
public boolean foo(boolean p, boolean q, boolean r) {
    return !p || (q || r);
}
```

(e) We prove that $p \to (q \vee r)$ is equivalent to $\neg q \to (q \to r)$ below:

$$\begin{aligned}
p \to (q \vee r) &\equiv \neg p \vee (q \vee r) && \text{Law of Implication} \\
&\equiv (\neg p \vee q) \vee r && \text{Associativity} \\
&\equiv (q \vee \neg p) \vee r && \text{Commutivity} \\
&\equiv q \vee (\neg p \vee r) && \text{Associativity} \\
&\equiv \neg\neg q \vee (\neg p \vee r) && \text{Double Negation} \\
&\equiv \neg q \to (p \to r) && \text{Law of Implication}
\end{aligned}$$

3. Simplification

$$\begin{aligned}
6(3p-2)-(5p+1)(2p+2) &= 6(3p-2)-10p^2+12p+2 && \text{Multiply right-most polynomials} \\
&= 18p-12-10p^2+12p+2 && \text{Multiply in the 6} \\
&= -10p^2+30p-10 && \text{Combine terms}
\end{aligned}$$

3. Sneak Peek

Suppose n is a positive integer. We must figure out if $n(n+1)(n+2)$ is evenly divisible by 6.

We know that n, $n+1$, and $n+2$ are three consecutive numbers. We also know that if we look at any two consecutive numbers, one of them must be divisible by 2. Likewise, if we look at any three consecutive numbers, one of them must be divisible by 3.

Therefore, since n, $n+1$, and $n+2$ are consecutive numbers, and we know at least one of them is divisible by 2 and another by 3, we can conclude that the product $n(n+1)(n+2)$ must contain the factors 2 and 3.

So, if the product contains those two factors, we also know that it must be evenly divisible by 6.

4. Yummy

First, number the mice from 0 to 4 and the batches from 0 to 29. Note that it is possible to write the numbers from 1 to 30 in binary using at most 5 bits since $2^5 = 32 \geq 30$. We feed mouse m a piece of cupcake from batch b if and only if the binary representation of b has a 1 for the bit corresponding to 2^m.

We then wait 23 hours and see which mice have died. To figure out which tray is tainted, we add the powers of two corresponding to each dead mouse together. The cupcake batch with the matching number is the poisoned one.

For clarity, we show an example with fewer mice and cupcakes in the table below.

	Mouse 0	**Mouse 1**	**Mouse 2**
Batch 0	X	X	X
Batch 1	X	X	
Batch 2	X		X
Batch 3	X		
Batch 4		X	X
Batch 5		X	
Batch 6			X
Batch 7			

A cell in the above table has an X in it precisely when the mouse in the column will eat from the batch in the row.

Notice that when any ingle batch is poisoned, a unique combination of mice will die. This means using just 3 mice, we can test up to 8 batches simultaneously.