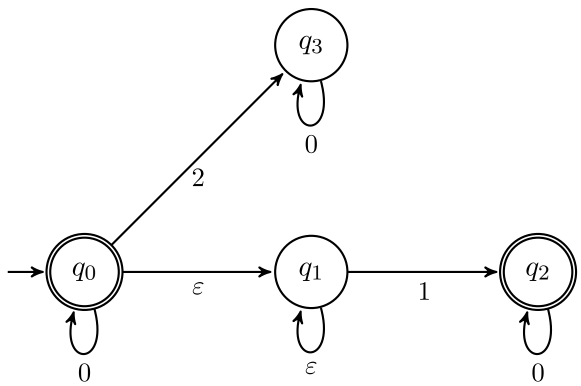
CSE 311: Foundations of Computing I

Section : NFAs, Minimization, Irregular Languages Solutions

0. NFAs

(a) What language does the following NFA accept?



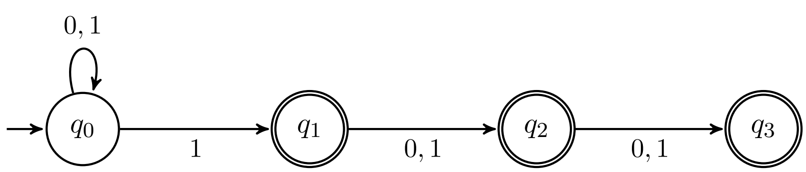
Solution:

All strings of only 0's and 1's not containing more than one 1.

(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

Solution:

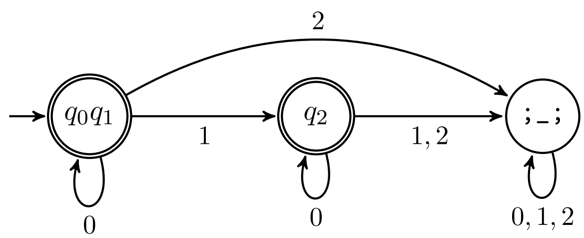
The following is one such NFA:



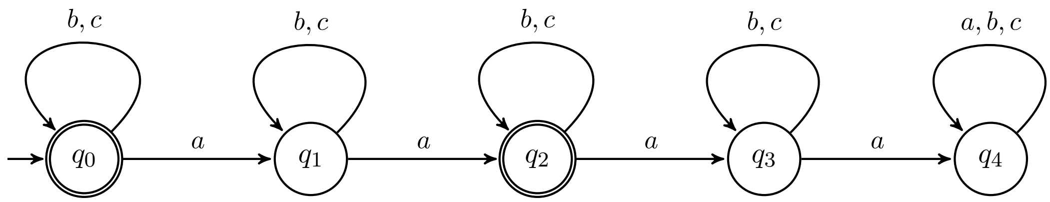
1. DFAs & Minimization

(a) Convert the NFA from 0a to a DFA, then minimize it.

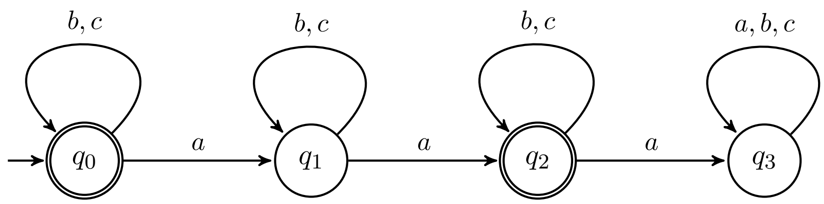
Solution:



(b) Minimize the following DFA:



Solution:



2. Irregularity

(a) Let $\Sigma = \{0, 1\}$. Prove that $\{0^n 1^n 0^n \ : \ n \geq 0\}$ is not regular.

Solution:

Let $L = \{0^n 1^n 0^n \ : \ n \geq 0\}$. Let D be an arbitrary DFA, and suppose for contradiction that D accepts L. Consider $S = \{0^n 1^n : n \geq 0\}$. Since S contains infinitely many strings and D has a finite number of states, two strings in S must end up in the same state. Say these strings are $0^i 1^i$ and $0^j 1^j$ for some $i, j \geq 0$ such that $i \neq j$. Append the string 0^i to both of these strings. The two resulting strings are:

$a = 0^i 1^i 0^i$ Note that $a \in L$.

$b = 0^j 1^j 0^i$ Note that $b \notin L$, since $i \neq j$.

Since a and b end up in the same state, but $a \in L$ and $b \notin L$, that state must be both an accept and reject state, which is a contradiction. Since D was arbitrary, there is no DFA that recognizes L, so L is not regular.

(b) Let $\Sigma = \{0, 1, 2\}$. Prove that $\{0^n (12)^m \ : \ n \geq m \geq 0\}$ is not regular.

Solution:

Let $L = \{0^n (12)^m \ : \ n \geq m \geq 0\}$. Let D be an arbitrary DFA, and suppose for contradiction that D accepts L. Consider $S = \{0^n : n \geq 0\}$. Since S contains infinitely many strings and D has a finite number of states, two strings in S must end up in the same state. Say these strings are 0^i and 0^j for some $i, j \geq 0$ such that $i > j$. Append the string $(12)^i$ to both of these strings. The two resulting strings are:

$a = 0^i (12)^i$ Note that $a \in L$.

$b = 0^j (12)^i$ Note that $b \notin L$, since $i > j$.

Since a and b end up in the same state, but $a \in L$ and $b \notin L$, that state must be both an accept and reject state, which is a contradiction. Since D was arbitrary, there is no DFA that recognizes L, so L is not regular.

(c) Let $\Sigma = \{(,)\}$. Prove that the language $\{s \in \Sigma^* \ : \ s \text{ is composed of correctly nested \& balanced parentheses}\}$ is not regular.

Solution:

Let $L = \{s \in \Sigma^* \ : \ s \text{ is composed of correctly nested \& balanced parentheses}\}$. Let D be an arbitrary DFA, and suppose for contradiction that D accepts L. Consider $S = \{(^n : n \geq 0\}$. Since S contains infinitely many strings and D has a finite number of states, two strings in S must end up in the same state. Say these strings are $(^i$ and $(^j$ for some $i, j \geq 0$ such that $i \neq j$. Append the string $)^i$ to both of these strings. The two resulting strings are:

$a = (^i)^i$ Note that $a \in L$.

$b = (^j)^i$ Note that $b \notin L$, since $i \neq j$, so the left and right parentheses are imbalanced.

Since a and b end up in the same state, but $a \in L$ and $b \notin L$, that state must be both an accept and reject state, which is a contradiction. Since D was arbitrary, there is no DFA that recognizes L, so L is not regular.