

CSE 311: Foundations of Computing I

QuickCheck: Induction Solutions (due Thursday, May 5)

0. Another Induction

Prove that $2^n \geq 5n$ for all integers $n \geq 5$ by induction.

Solution:

Let $P(n)$ be " $2^n \geq 5n$ ". We will prove $P(n)$ for all integers $n \geq 5$ by induction.

Base Case ($n = 5$): $2^5 = 32 \geq 25 = 5 \cdot 5$, so $P(5)$ holds.

Inductive Hypothesis: Assume that $2^j \geq 5j$ for an arbitrary integer $j \geq 5$.

Inductive Step: Goal: Show $2^{j+1} \geq 5(j+1)$

Now

$$\begin{aligned} 2^{j+1} &= 2 \cdot 2^j \\ &\geq 2 \cdot 5j && \text{[By IH]} \\ &\geq 5j + 5j \\ &\geq 5j + 5 \cdot 1 && \text{[Since } j \geq 5 > 1\text{]} \\ &\geq 5(j+1) \end{aligned}$$

So $P(j) \rightarrow P(j+1)$ for an arbitrary integer $j \geq 5$.

Conclusion: $P(n)$ holds for all integers $n \geq 5$ by induction.