CSE 311: Foundations of Computing I

Section 3: FOL and Inference

0. Formal Proofs

For this question only, write *formal proofs*.

(a) Prove $\forall x\ (R(x) \wedge S(x))$ given $\forall x\ (P(x) \rightarrow (Q(x) \wedge S(x)))$, and $\forall x\ (P(x) \wedge R(x))$.

(b) Prove $\exists x\ \neg R(x)$ given $\forall x\ (P(x) \vee Q(x))$, $\forall x\ (\neg Q(x) \vee S(x))$, $\forall x\ (R(x) \rightarrow \neg S(x))$, and $\exists x\ \neg P(x)$.

1. Odds and Ends

Prove that for any even integer, there exists an odd integer greater than that even integer.

2. Magic Squares

Prove that if a real number $x \neq 0$, then $x^2 + \dfrac{1}{x^2} \geq 2$.

3. Primality Checking

When brute forcing if the number p is prime, you only need to check possible factors up to $\sqrt{p}$. In this problem, you'll prove why that is the case. Prove that if $n = ab$, then either a or b is at most $\sqrt{n}$.
(*Hint:* You want to prove an implication; so, start by assuming $n = ab$. Then, continue by writing out your assumption for contradiction.)

4. Even More Negative

Show that $\forall (x \in \mathbb{Z}).\ (\text{Even}(x) \rightarrow (-1)^x = 1)$

5. That's Odd...

Prove that every odd natural number can be expressed as the difference between two consecutive perfect squares.

6. United We Stand

We say that a set S is closed under an operation $\star$ iff $\forall (x, y \in S).\ (x \star y \in S)$.

(a) Prove $\mathbb{Z}$ is closed under $-$.

(b) Prove that $\mathbb{Z}$ is *not* closed under $/$.

(c) Prove that $\mathbb{I}$ is *not* closed under $+$.

7. A Hint of Things to Come

Prove that $\forall (a, b \in \mathbb{Z}).\ a^2 - 4b \neq 2$.

8. Proofs or it didn't happen!

(a) Prove that if x is an odd integer and y is an integer, then xy is odd if and only if y is odd.

(b) Prove that for integers x and y, if $(x + y)^2 = 16$ that $xy < 10$.

(c) Prove that for positive integers x, a where x is odd, there is an even integer y such that $a^x \leq a^y$.