CSE 311: Foundations of Computing I

QuickCheck: FOL and Inference Solutions

0. Oddly Even

Let Even(x) be $\exists y\ (x = 2y)$, and let Odd(x) be $\exists y\ (x = 2y + 1)$. Let the domain of discourse be the set of all integers.

(a) Translate the following statement into English.

$$\forall x\ \forall y\ ((\text{Odd}(x) \land \text{Odd}(y)) \to \text{Even}(x + y))$$

Solution:

For all integers x, y, if x and y are odd, then $x + y$ is even.

(b) Prove the statement from part (a) using an *English proof*.

Solution:

Let x and y be arbitrary odd integers. Then, $x = 2k_x + 1$ and $y = 2k_y + 1$ for some $k_x, k_y \in \mathbb{Z}$ by the definition of odd. Combining the equations, we see: $x + y = 2k_x + 1 + 2k_y + 1 = 2(k_x + k_y + 1)$. So, $x + y$ is even, by definition of even.

In case you were curious, we can also prove this using formal notation. Note that this proof is just as precise and rigorous as our English proof, but is longer and harder to follow:

1. Let x be an arbitrary integer.
2. Let y be an arbitrary integer.
   - 3.1. Odd(x) $\land$ Odd(y) [Assumption]
   - 3.2. Odd(x) [Elim $\land$: 3.1]
   - 3.3. $\exists k\ (x = 2k + 1)$ [Definition of Odd, 3.2]
   - 3.4. $x = 2k + 1$ [Elim $\exists$: 3.3]
   - 3.5. Odd(y) [Elim $\land$: 3.1]
   - 3.6. $\exists k\ (y = 2k + 1)$ [Definition of Odd, 3.5]
   - 3.7. $y = 2j + 1$ [Elim $\exists$: 3.7]
   - 3.8. $x + y = 2k + 1 + 2j + 1$ [Algebra: 3.4, 3.7]
   - 3.9. $x + y = 2(k + j + 1)$ [Algebra: 3.8]
   - 3.10. $\exists r\ (x + y = 2r)$ [Intro $\exists$: 3.9]
   - 3.11. Even($x + y$) [Definition of Even, 3.10]
3. Odd(x) $\land$ Odd(y) $\to$ Even($x + y$) [Direct Proof Rule]