

CSE 311: Foundations of Computing I

QuickCheck: NFAs, Minimization, Irregular Languages Solutions (due Thursday, May 26)

0. Irregularity

Let $\Sigma = \{0, 1\}$. Prove that the set $\{ww : w \in \Sigma^*\}$ is irregular.

Solution:

Let $L = \{ww : w \in \Sigma^*\}$. Let D be an arbitrary DFA, and suppose for contradiction that D accepts L . Consider $S = \{10^n : n \geq 0\}$. Since S contains infinitely many strings and D has a finite number of states, two strings in S must end up in the same state. Say these strings are 10^i and 10^j for some $i, j \geq 0$ such that $i \neq j$. Append the string 10^i to both of these strings. The two resulting strings are:

$a = 10^i10^i$ Note that $a \in L$, since we can choose $w = 10^i$ and write a as ww .

$b = 10^j10^i$ Note that $b \notin L$, since the first half of b starts with 1 but the second half cannot.

Since a and b end up in the same state, but $a \in L$ and $b \notin L$, that state must be both an accept and reject state, which is a contradiction. Since D was arbitrary, there is no DFA that recognizes L , so L is not regular.