CSE 311: Foundations of Computing I

Section : More Midterm Review

0. Dividing by Nines

Prove that $9 \mid n^3 + (n+1)^3 + (n+2)^3$ for all $n > 1$ by induction.

1. Those 2's Just Grow Up So Fast

Prove that $6n + 6 < 2^n$ for all $n \geq 6$.

2. Proof by Harmonicas

Define

$$H_i = 1 + \frac{1}{2} + \cdots + \frac{1}{i}$$

Prove that $H_{2^n} \geq 1 + \frac{n}{2}$ for $n \in \mathbb{N}$.

3. Odds and Ends

Prove that for any even integer, there exists an odd integer greater than that even integer.

4. Magic Squares

Prove that if a real number $x \neq 0$, then $x^2 + \dfrac{1}{x^2} \geq 2$.

5. Primality Checking

When brute forcing if the number p is prime, you only need to check possible factors up to $\sqrt{p}$. In this problem, you'll prove why that is the case. Prove that if $n = ab$, then either a or b is at most $\sqrt{n}$.

6. Even More Negative

Show that $\forall (x \in \mathbb{Z}). (\text{Even}(x) \rightarrow (-1)^x = 1)$

7. That's Odd...

Prove that every odd natural number can be expressed as the difference between two consecutive perfect squares.

8. United We Stand

We say that a set S is closed under an operation $\star$ iff $\forall (x, y \in S). (x \star y \in S)$.

(a) Prove $\mathbb{Z}$ is closed under $-$.

(b) Prove that $\mathbb{Z}$ is *not* closed under $/$.

(c) Prove that $\mathbb{I}$ is *not* closed under $+$.

9. A Hint of Things to Come

Prove that $\forall (a, b \in \mathbb{Z}).\ a^2 - 4b \neq 2$.

10. Proofs or it didn't happen!

(a) Prove that if x is an odd integer and y is an integer, then xy is odd if and only if y is odd.

(b) Prove that for integers x and y, if $(x+y)^2 = 16$ that $xy < 10$.

(c) Prove that for positive integers x, a where x is odd, there is an even integer y such that $a^x \leq a^y$.

11. To B or not to B

Prove $(A \setminus B) \cap B = \varnothing$.