

CSE 311: Foundations of Computing I

Section : Sets and Modular Arithmetic

0. How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say so.

- (a) $A = \{1, 2, 3, 2\}$
- (b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \dots\}$
- (c) $C = A \times (B \cup \{7\})$
- (d) $D = \emptyset$
- (e) $E = \{\emptyset\}$
- (f) $F = \mathcal{P}(\{\emptyset\})$

1. Set = Set

Prove the following set identities.

- (a) Let the universal set be \mathcal{U} . Prove $\overline{\overline{X}} = X$ for any set X .
- (b) Prove $(A \oplus B) \oplus B = A$ for any sets A, B .
- (c) Prove $A \cup B \subseteq A \cup B \cup C$ for any sets A, B, C .
- (d) Let the universal set be \mathcal{U} . Prove $A \cap \overline{B} \subseteq A \setminus B$ for any sets A, B .

2. Modular Arithmetic

- (a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.
- (b) Prove that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv_m b$, where a and b are integers, then $a \equiv_n b$.

3. New Definitions

- We say (\mathcal{M}, \star) is a *magma* iff $\forall (x \in \mathcal{M}) \forall (y \in \mathcal{M}) x \star y \in \mathcal{M}$.
 - We say “ e is a *left-identity*” in a magma (\mathcal{M}, \star) , iff $\forall (a \in \mathcal{M}) e \star a = a$.
 - We say “ e is a *right-identity*” in a magma (\mathcal{M}, \star) , iff $\forall (a \in \mathcal{M}) a \star e = a$.
 - We say “ x^{-1} is a *right-inverse of x* ” in a magma (\mathcal{M}, \star) , iff for all right-identities, e , in \mathcal{M} , $x \star x^{-1} = e$.
- (a) Let (\mathcal{Q}, Δ) be a magma. Prove that if a and b are both right-identities and all $m \in \mathcal{Q}$ have right-inverses, then $a = b$.
 - (b) Let (\mathcal{R}, \square) be an associative magma with a left and right identity $e \in \mathcal{R}$. Prove that for all $a \in \mathcal{R}$, if a has a right inverse a^{-1} , then $(a^{-1})^{-1} = a$.