CSE 311: Foundations of Computing I

QuickCheck: Sets Solutions

0. Sets All Folks!

Prove $(\mathcal{P}(A) = \mathcal{P}(B)) \to (A = B)$ using an English proof. Remember that $\mathcal{P}(X) = \{x : x \subseteq X\}$.

Solution:

Let A and B be arbitrary sets. Suppose that $\mathcal{P}(A) = \mathcal{P}(B)$. We will prove that $A = B$.

Note that $A \subseteq A$, by definition of subset equality. This means, by definition of power set, that $A \in \mathcal{P}(A)$. Since $\mathcal{P}(A) = \mathcal{P}(B)$ by our assumption, $A \in \mathcal{P}(B)$. By definition of power set, this means that $A \subseteq B$.

Next, note that $B \subseteq B$, by definition of subset equality. This means, by definition of power set, that $B \in \mathcal{P}(B)$. Since $\mathcal{P}(A) = \mathcal{P}(B)$ by our assumption, $B \in \mathcal{P}(A)$. By definition of power set, this means that $B \subseteq A$.

As $A \subseteq B$ and $B \subseteq A$, by theorem of subset containment, $A = B$.

Thus, $(\mathcal{P}(A) = \mathcal{P}(B)) \to (A = B)$.