CSE 311: Foundations of Computing I

Section 3: FOL and Inference Solutions

0. Translate to Logic

Express each of these English sentences into logical expressions using predicates, quantifiers, and logical connectives. For each sentence, also define an appropriate domain of discourse. Use the constant "Sam" as the single person that is Sam. You may not use any other constants in your expression. *Note*: In the context of these statements, a person cannot be friends with themselves.

After you have found a logical expression for each sentence, negate and simplify the entire expression, pushing negation symbols as far in as possible; any negation symbol should be directly in front of a predicate.

(a) Everybody who is friends with Sam dislikes all of Sam's friends.

Solution:

Let the domain be all people. Let Friend(x, y) be "x is a friend of y" and Dislikes(x, y) be "x dislikes y".

$$\forall x\ (\text{Friend}(x, \text{Sam}) \to \forall y\ (\text{Friend}(y, \text{Sam}) \to \text{Dislikes}(x, y)))$$

Negation:

$$\exists x\ (\text{Friend}(x, \text{Sam}) \wedge \exists y\ (\text{Friend}(y, \text{Sam}) \wedge \neg\text{Dislikes}(x, y))$$

(b) Everybody who is friends with Sam dislikes all of Sam's other friends.

Solution:

Let the domain be all people. Let Equal(x, y) be "x equals y", Friend(x, y) be "x is a friend of y" and Dislikes(x, y) be "x dislikes y".

$$\forall x\ (\text{Friend}(x, \text{Sam}) \to \forall y\ ((\text{Friend}(y, \text{Sam}) \wedge \neg\text{Equal}(x, y)) \to \text{Dislikes}(x, y)))$$

Negation:

$$\exists x\ (\text{Friend}(x, \text{Sam}) \wedge \exists y; (\text{Friend}(y, \text{Sam}) \wedge \neg\text{Equal}(x, y) \wedge \neg\text{Dislikes}(x, y)))$$

(c) One of Sam's friends is friends with all of Sam's other friends.

Solution:

Let the domain be all people. Let Equal(x, y) be "x equals y", and Friend(x, y) be "x is a friend of y".

$$\exists x\ (\text{Friend}(x, \text{Sam}) \wedge \forall y\ ((\text{Friend}(y, \text{Sam}) \wedge \neg\text{Equal}(x, y)) \to \text{Friend}(x, y)))$$

Negation:

$$\forall x\ (\neg\text{Friend}(x, \text{Sam}) \vee \exists y\ (\text{Friend}(y, \text{Sam}) \wedge \neg\text{Equal}(x, y) \wedge \neg\text{Friend}(x, y)))$$

(d) Any friend of Sam taking the same class as Sam is friends with one of Sam's friends who shares none of his classes.

Solution:

Let the domain be all people. Let Equal(x, y) be "x equals y", Friend(x, y) be "x is a friend of y", and SharesClass(x, y) be "x shares a class with y".

$$\forall x\ ((\text{Friend}(x, \text{Sam}) \land \text{SharesClass}(x, \text{Sam})) \to \exists y\ (\text{Friend}(y, \text{Sam}) \land$$
$$\neg\text{Equal}(x, y) \land \text{Friend}(x, y) \land \neg\text{SharesClass}(y, \text{Sam})))$$

Negation:

$$\exists x\ ((\text{Friend}(x, \text{Sam}) \land \text{SharesClass}(x, \text{Sam}) \land$$
$$\forall y\ (\neg\text{Friend}(y, \text{Sam}) \lor \text{Equal}(x, y) \lor \neg\text{Friend}(x, y) \lor \text{SharesClass}(y, \text{Sam})))$$

1. Formal Proofs

For this question only, write *formal proofs*.

(a) Prove $\forall x\ (R(x) \land S(x))$ given $\forall x\ (P(x) \to (Q(x) \land S(x)))$, and $\forall x\ (P(x) \land R(x))$.

Solution:

1.	Let x be arbitrary.	
2.	$\forall x\ (P(x) \land R(x))$	[Given]
3.	$P(x) \land R(x)$	[Elim $\forall$: 2]
4.	$P(x)$	[Elim $\land$: 3]
5.	$R(x)$	[Elim $\land$: 3]
6.	$\forall x\ (P(x) \to (Q(x) \land S(x)))$	[Given]
7.	$P(x) \to (Q(x) \land S(x))$	[Elim $\forall$: 6]
8.	$Q(x) \land S(x)$	[MP: 4, 7]
9.	$S(x)$	[Elim $\land$: 8]
10.	$R(x) \land S(x)$	[Intro $\land$: 5, 9]
11.	$\forall x\ (R(x) \land S(x))$	[Intro $\forall$: 10]

(b) Prove $\exists x\ \neg R(x)$ given $\forall x\ (P(x) \lor Q(x))$, $\forall x\ (\neg Q(x) \lor S(x))$, $\forall x\ (R(x) \to \neg S(x))$, and $\exists x\ \neg P(x)$.

Solution:

1.	$\exists x\ \neg P(x)$	[Given]
2.	$\neg P(c)$	[Elim $\exists$: 1]
3.	$\forall x\ (P(x) \vee Q(x))$	[Given]
4.	$P(c) \vee Q(c)$	[Elim $\forall$: 3]
5.	$Q(c)$	[Elim $\vee$: 2, 4]
6.	$\forall x\ (\neg Q(x) \vee S(x))$	[Given]
7.	$\neg Q(c) \vee S(c)$	[Elim $\forall$: 6]
8.	$S(c)$	[Elim $\vee$: 5, 7]
9.	$\forall x\ (R(x) \to \neg S(x))$	[Given]
10.	$R(c) \to \neg S(c)$	[Elim $\forall$: 9]
11.	$\neg\neg S(c) \to \neg R(c)$	[Contrapositive: 10]
12.	$S(c) \to \neg R(c)$	[Double Negation: 11]
13.	$\neg R(c)$	[MP: 8, 12]
14.	$\exists x\ \neg R(x)$	[Intro $\exists$: 13]