CSE 311: Foundations of Computing I

Section 2: Equivalences and Predicate Logic

0. Equivalences

Prove that each of the following pairs of propositional formulae are equivalent using propositional equivalences.

(a) $(p \rightarrow q) \wedge (q \rightarrow p)$ $\qquad$ $(p \wedge q) \vee (\neg p \wedge \neg q)$

(b) $\neg p \rightarrow (q \rightarrow r)$ $\qquad$ $q \rightarrow (p \vee r)$

1. Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent by finding inputs they differ on.

(a) $p \rightarrow q$ $\qquad$ $q \rightarrow p$

(b) $p \rightarrow (q \wedge r)$ $\qquad$ $(p \rightarrow q) \wedge r$

2. Boolean Algebra

For each of the following parts, write the logical expression using boolean algebra operators. Then, simplify it using axioms and theorems of boolean algebra.

(a) $\neg p \vee (\neg q \vee (p \wedge q))$

(b) $\neg(p \vee (q \wedge p))$

3. Canonical Forms

Consider the boolean functions $F(A, B, C)$ and $G(A, B, C)$ specified by the following truth table:

A	B	C	$F(A, B, C)$	$G(A, B, C)$
1	1	1	1	0
1	1	0	1	1
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1
0	1	0	1	0
0	0	1	0	1
0	0	0	1	0

(a) Write the DNF and CNF expressions for $F(A, B, C)$.

(b) Write the DNF and CNF expressions for $G(A, B, C)$.

4. Translate to Logic

Express each of these English sentences into logical expressions using predicates, quantifiers, and logical connectives.

(a) A cuttlefish is poisonous only if it has spots and does not eat shrimp.

(b) Every sea creature has either fins or a shell, but not both, unless the sea creature is a cuttlefish.

(c) When a shark meets a fish, that shark will eat that fish only if it is not also a shark.

5. Translate to English

Translate these system specifications into English where $F(p)$ is "Printer p is out of service", $B(p)$ is "Printer p is busy", $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued". Let the domain be all printers and print jobs.

(a) $\exists p\ (F(p) \wedge B(p)) \rightarrow \exists j\ L(j)$

(b) $(\forall p\ B(p)) \rightarrow (\exists j\ Q(j))$

(c) $\exists j\ (Q(j) \wedge L(j)) \rightarrow \exists p\ F(p)$

(d) $(\forall p\ B(p) \wedge \forall j\ Q(j)) \rightarrow \exists j\ L(j)$

6. Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other (or if they are equivalent).

(a) $\forall x\ \forall y\ P(x, y)$ $\qquad$ $\forall y\ \forall x\ P(x, y)$

(b) $\exists x\ \exists y\ P(x, y)$ $\qquad$ $\exists y\ \exists x\ P(x, y)$

(c) $\forall x\ \exists y\ P(x, y)$ $\qquad$ $\forall y\ \exists x\ P(x, y)$

(d) $\forall x\ \exists y\ P(x, y)$ $\qquad$ $\exists x\ \forall y\ P(x, y)$

7. Positively Different

For $a \in \mathbb{R}$ and $f : \mathbb{R} \rightarrow \mathbb{R}$, show that (a) and (b) have different meanings.

(a) $\forall y\ ((y > 0) \rightarrow \exists z\ ((z > 0) \wedge ((|x - a| < z) \rightarrow (|f(x) - f(a)| < y)))$

(b) $\exists z\ ((z > 0) \wedge \forall y\ ((y > 0) \rightarrow ((|x - a| < z) \rightarrow (|f(x) - f(a)| < y)))$

8. TR∀NSL∀TOR

Express each of these sentences using predicates, quantifiers, and logical connectives. Make sure to define a domain for each part.

(a) There are at least two fluffy dogs in every happy house.

(b) If there is a new book or a cheap book by my favorite author in the bookstore, then I will buy it.

(c) All parks have at least one duck pond with more than one duck.