CSE 311: Foundations of Computing I

QuickCheck: Logic Solutions

0. Fool Me Once...

Consider the following sentence:

I only left the light on in my office if I rushed home, but I either rushed home or left early.

(a) Is the sentence a proposition? Explain why or why not.

Solution:

This sentence is a proposition. It has a truth value associated to it (i.e. it can be shown to be a true or false statement) and is well-formed (grammatically correct, unambiguous).

(b) Define smaller propositions in preparation for converting the sentence to logical notation.

Solution:

p: I left the light on in my office q: I rushed home

- $r: \mathsf{I}$ left early
- (c) Convert the sentence to logical notation.

Solution:

 $(p \to q) \land (q \oplus r)$

Something that confused a lot of people was the ordering of p and q in the logical expression. The proposition states that the light was ONLY left on if I rushed home early.

However, I could rush home early and not leave the light on. The causal relationship implied is then that if the light is left on in my office, I must have rushed home.

If you're still having trouble with this, build a truth table for statements $p \rightarrow q$ and $q \rightarrow p$ and check to see when the statement will be violated based on the truth table solution to an implication (See Lecture 1 notes).

(d) Fill in the following truth table for the sentence:

Solution:

p	q	r	$p \rightarrow q$	$q\oplus r$	$(p \to q) \land (q \oplus r)$
Т	Т	Т	Т	F	F
Т	Т	F	Т	Т	Т
Т	F	Т	F	Т	F
Т	F	F	F	F	F
F	Т	Т	Т	F	F
F	Т	F	Т	Т	Т
F	F	Т	Т	Т	Т
F	F	F	Т	F	F