CSE 311: Foundations of Computing I

QuickCheck: Number Theory Solutions (due Thursday, April 28)

0. Extended Euclidian Algorithm

Find the multiplicative inverse y of 7 mod 33. That is, find y such that $7y \equiv 1 \pmod{33}$. You should use the extended Euclidean Algorithm. Your answer should be in the range $0 \leq y < 33$.

Solution:

First, we find the gcd:

$$\begin{aligned}
\gcd(33,7) &= \gcd(7,5) & 33 &= \boxed{7} \bullet 4 + 5 & (1)\\
&= \gcd(5,2) & 7 &= \boxed{5} \bullet 1 + 2 & (2)\\
&= \gcd(2,1) & 5 &= \boxed{2} \bullet 2 + 1 & (3)\\
&= \gcd(1,0) & 2 &= 1 \bullet 2 + 0 & (4)\\
&= 1 & & & (5)
\end{aligned}$$

Next, we re-arrange equations (1) - (3) by solving for the remainder:

$$\begin{aligned}
1 &= 5 - \boxed{2} \bullet 2 & (6)\\
2 &= 7 - \boxed{5} \bullet 1 & (7)\\
5 &= 33 - \boxed{7} \bullet 4 & (8)\\
& & (9)
\end{aligned}$$

Now, we backward substitute into the boxed numbers using the equations:

$$\begin{aligned}
1 &= 5 - \boxed{2} \bullet 2\\
&= 5 - (7 - \boxed{5} \bullet 1) \bullet 2\\
&= 3 \bullet \boxed{5} - 7 \bullet 2\\
&= 3 \bullet (33 - \boxed{7} \bullet 4) - 7 \bullet 2\\
&= 33 \bullet 3 + 7 \bullet -14
\end{aligned}$$

So, $1 = 33 \bullet 3 + \boxed{7} \bullet -14$. Thus, $33 - 14 = 19$ is the multiplicative inverse of 7 mod 33.