CSE 311: Foundations of Computing I

Section : Sets and Modular Arithmetic

0. How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say .

(a) $A = \{1, 2, 3, 2\}$

(b) $B = \{\{\},\ \{\{\}\},\ \{\{\}, \{\}\},\ \{\{\}, \{\}, \{\}\},\ \ldots\}$

(c) $C = A \times (B \cup \{7\})$

(d) $D = \varnothing$

(e) $E = \{\varnothing\}$

(f) $F = \mathcal{P}(\{\varnothing\})$

1. Set = Set

Prove the following set identities.

(a) Let the universal set be $\mathcal{U}$. Prove $\overline{\overline{X}} = X$ for any set X.

(b) Prove $(A \oplus B) \oplus B = A$ for any sets A, B.

(c) Prove $A \cup B \subseteq A \cup B \cup C$ for any sets A, B, C.

(d) Let the universal set be $\mathcal{U}$. Prove $A \cap \overline{B} \subseteq A \setminus B$ for any sets A, B.

2. Casting Out Nines

Let $n \in \mathbb{N}$. Prove that if $n \equiv 0 \pmod 9$, then the sum of the digits of n is a multiple of 9.
You may use without proof that $a \equiv b \pmod m \rightarrow a^i \equiv b^i \pmod m$.

3. Modular Arithmetic

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

(b) Prove that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv b \pmod m$, where a and b are integers, then $a \equiv b \pmod n$.