

CSE 311: Foundations of Computing I

QuickCheck: FOL and Inference Solutions (due Thursday, April 14)

0. Oddly Even

Let $\text{Even}(x)$ be $\exists y x = 2y$, and let $\text{Odd}(x)$ be $\exists y x = 2y + 1$. Let the domain of discourse be the set of all integers.

- (a) Translate the statement

$$\forall x \forall y ((\text{Odd}(x) \wedge \text{Odd}(y)) \rightarrow \text{Even}(x + y))$$

into English.

Solution:

For all integers x, y , if x and y are odd, then $x + y$ is even.

- (b) Prove the statement from part (a) using a *formal proof*.

1. Let x be an integer.
2. Let y be an integer.

Solution:

1. Let x be an integer.
2. Let y be an integer.
 - 3.1. $\text{Odd}(x) \wedge \text{Odd}(y)$ [Assumption]
 - 3.2. $\text{Odd}(x)$ [Elim \wedge : 3.1]
 - 3.3. $\exists k x = 2k + 1$ [Definition of Odd, 3.2]
 - 3.4. $x = 2k + 1$ [Elim \exists : 3.3]
 - 3.5. $\text{Odd}(y)$ [Elim \wedge : 3.1]
 - 3.6. $\exists k y = 2k + 1$ [Definition of Odd, 3.5]
 - 3.7. $y = 2j + 1$ [Elim \exists : 3.7]
 - 3.8. $x + y = 2k + 1 + 2j + 1$ [Algebra: 3.4, 3.7]
 - 3.9. $x + y = 2(k + j + 1)$ [Algebra: 3.8]
 - 3.10. $\exists r x + y = 2r$ [Intro \exists : 3.9]
 - 3.11. $\text{Even}(x + y)$ [Definition of Even, 3.10]
3. $\text{Odd}(x) \wedge \text{Odd}(y) \rightarrow \text{Even}(x + y)$ [Direct Proof Rule]

Or, as an English proof:

Let x and y be arbitrary odd integers. Then, $x = 2k_x + 1$ and $y = 2k_y + 1$ for some $k_x, k_y \in \mathbb{Z}$. Combining the equations, we see: $x + y = 2k_x + 1 + 2k_y + 1 = 2(k_x + k_y + 1)$. So, $x + y$ is even, by definition of even.