CSE 311: Foundations of Computing I

QuickCheck: Gates and Equivalence Solutions (due Thursday, October 2)

0. If you turn the paper horizontally, the circuits look like robots (:

(a) Convert each of the following circuits to logical expressions.



Solution:

(i)
$$((\neg p) \land (p \lor q)) \land \neg \neg q$$

- (ii) $\neg p \land (q \land q)$
- (b) Prove that (i) and (ii) are equivalent using a truth table.

Solution:

р	q	$(\neg p \land (p \lor q))$	$(\neg p \land (p \lor q)) \land \neg \neg q$	$\neg p \land (q \land q)$
Т	Т	F	F	F
Т	F	F	F	F
F	Т	Т	Т	Т
F	F	F	F	F

(c) Prove that (i) and (ii) are equivalent using propositional equivalences. See your handout for a full list of them.

Solution:

$$\begin{array}{ll} (\neg p \land (p \lor q)) \land \neg \neg q \equiv (\neg p \land (p \lor q)) \land q & [\text{Double Negation}] \\ \equiv \neg p \land ((p \lor q) \land q) & [\text{Associative}] \\ \equiv \neg p \land (q \land (q \lor p)) & [\text{Commutativity (twice)}] \\ \equiv \neg p \land q & [\text{Absorbtion}] \\ \equiv \neg p \land (q \land q) & [\text{Idempotency}] \end{array}$$