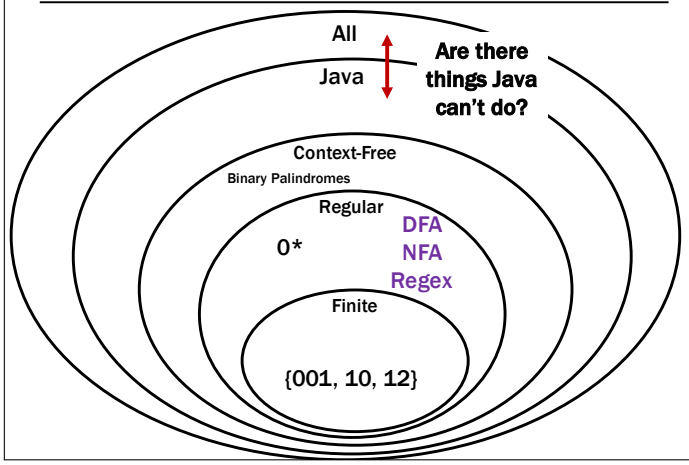
CSE 311: Foundations of Computing

Lecture 25: Limits of Computation

```
DEFINE DOESITHALT(PROGRAM):
{
    RETURN TRUE;
}
```

THE BIG PICTURE SOLUTION TO THE HALTING PROBLEM

Languages and Machines!



What We're About To Do....

Today, we will dispel the notion that Java is a magical language that allows us to solve any problem we want if we're smart enough.

An Assignment Too Simple for 142!

Students should write a Java program that...

- Prints "Hello" to the console
- Eventually exits

GradeIt, PracticeIt, etc. need to grade the students.

How do we write that grading program?

Follow Up Question

What does this program do?

```
_(__,___,____){___/__<=1?_(__,___+1,___
_):!(___%__)?_(__,___+1,0):___%__==___
/__&&!____?(printf("%d\t",___/__),_(__,_
__+1,0)):___%__>1&&___%__<___/__?_( __,1+
___,____+!(___/__%(___%__))):___<__*__
?_(__,___+1,____):0;}main(){_(100,0,0);}
```

Follow Up Question

```
public static int collatz(n) {
   if (n == 1) {
      return 1;
   }
   if (n % 2 == 0) {
      return collatz(n/2)
   }
   else {
      return collatz(3n + 1)
   }
}
```

What is in the set { x : collatz(n) = 1}?

Some Notation and Starting Ideas

We're going to be talking about *Java code* a lot.

CODE(**P**) will mean "the code of the program P"

So, consider the following function:

```
public String P(String x) {
   return new String(Arrays.sort(x.toCharArray());
}
```

What is **P**(CODE(**P**))?

"((()))..;AACPSSaaabceeggghiiiilnnnnnoopprrrrrrrrrrssstttttttuuwxxyy{}"

The Halting Problem

Given:

– CODE(**P**) for a program **P**

Output:

– **true** if **P** halts

– **false** if **P** does not halt

The "standard" version of the halting problem takes some number as input. We consider this one, because it's easier to think about.

Proof Strategy

Remember, this means X is a program and HALT(X) is true when X halts and false otherwise.

Imagine we had a HALT(X) function which solved the halting problem…

Our goal is to write a program that CONFUSES the function HALT so that it does the wrong thing.

```
public static void PROGRAM() {




}
```

Proof Strategy

Remember, this means X is a program and HALT(X) is true when X halts and false otherwise.

Imagine we had a HALT(X) function which solved the halting problem…

Our goal is to write a program that CONFUSES the function HALT so that it does the wrong thing.

```
public static void PROGRAM() {
   if (/* I should halt */) {
      /* don't halt */
   }
   else {
      /*    halt    */
   }
}
```

Proof Strategy

Remember, this means X is a program and HALT(X) is true when X halts and false otherwise.

Imagine we had a HALT(X) function which solved the halting problem…

Our goal is to write a program that CONFUSES the function HALT so that it does the wrong thing.

```
public static void PROGRAM() {
   if (/* I should halt */) {
      while (true);
   }
   else {
      return;
   }
}
```

Proof Strategy

Remember, this means X is a program and HALT(X) is true when X halts and false otherwise.

Imagine we had a HALT(X) function which solved the halting problem…

Our goal is to write a program that CONFUSES the function HALT so that it does the wrong thing.

```
public static void PROGRAM() {
   if (HALT(MY_SOURCE_CODE)) {
      while (true);
   }
   else {
      return;
   }
}
```

Proof Strategy

Remember, this means X is a program and HALT(X) is true when X halts and false otherwise.

Suppose for contradiction we had a HALT(X) function which solved the halting problem…

```
public static void P(String input) {
   if (HALT(input)) {
      while (true);
   }
   else {
      return;
   }
}
```

Quick Question. What does this do?

OnMySourceCodeGENERATOR(P)

Proof Strategy

Remember, this means X is a program and HALT(X) is true when X halts and false otherwise.

Suppose for contradiction we had a HALT(X) function which solved the halting problem…

```
public static void P(String input) {
   if (HALT(input)) {
      while (true);
   }
   else {
      return;
   }
}
```

Does POnMySourceCode halt?

```
public static void HALT(String input) {
   // We don't know how this works,
   // but we assume that it does.

   // So, if input is a program that
   // halts, then this returns true.
   // Otherwise, it returns false.
}
```

```
void P(String input) {
   if (HALT(input)) {
     while (true);
   }
   else {
     return;
   }
}
```

Does POnMySourceCode halt?

Recall that POnMySourceCode does the same thing as P(CODE(POnMySourceCode)).

```
void POnMySourceCode() {
   if (HALT(CODE(POnMySourceCode))) {
        while (true);
   }
   else {
     return;
   }
}
```

```
public static void HALT(String input) {
   // We don't know how this works,
   // but we assume that it does.

   // So, if input is a program that
   // halts, then this returns true.
   // Otherwise, it returns false.
}
```

Suppose POnMySourceCode **halts.**

HALT(CODE(POnMySourceCode)) is true.

```
void POnMySourceCode() {
   if (HALT(CODE(POnMySourceCode))) {
        while (true);
   }
   else {
     return;
   }
}
```

So, this if statement is true!

So, the code loops forever!

This is a contradiction, so POnMySourceCode **does not halt.**

```
public static void HALT(String input) {
   // We don't know how this works,
   // but we assume that it does.

   // So, if input is a program that
   // halts, then this returns true.
   // Otherwise, it returns false.
}
```

Suppose POnMySourceCode **does not halt.**

HALT(CODE(POnMySourceCode)) is false.

```
void POnMySourceCode() {
   if (HALT(CODE(POnMySourceCode))) {
        while (true);
   }
   else {
     return;
   }
}
```

So, this if statement is false!

So, the code halts!

This is a contradiction, so POnMySourceCode **can't not halt.**

Suppose for contradiction we had a HALT(X) function which solved the halting problem…

Suppose POnMySourceCode **halts.**

Then, HALT(CODE(POnMySourceCode)) is true.

So, the if statement in POnMySourceCode is true!

So, the code loops forever!

This is a contradiction, so POnMySourceCode **does not halt.**

Suppose POnMySourceCode **does not halt.**

Then, HALT(CODE(POnMySourceCode)) is false.

So, the if statement in POnMySourceCode is true!

So, the code loops forever!

This is a contradiction, so POnMySourceCode **can't not halt.**

So, POnMySourceCode. So, P does not exist. So, HALT does not exist.

That's it!

- We proved that there is no Java program that can solve the Halting Problem.
- This tells us that there is no compiler that can check our programs and guarantee to find any infinite loops they might have.

~~That's it!~~

BUT WAIT…

THERE'S MORE!

```
public static void D(String input) {
   // Returns true if, when run, input
   // does X.
   // Otherwise, it returns false.
}
```

```
void P(String input) {
   if (D(input)) {
     D_IS_FALSE();
   }
   else {
     D_IS_TRUE();
   }
}
```

Is D(POnMySourceCode)true?

Recall that POnMySourceCode does the same thing as P(CODE(POnMySourceCode)).

```
void POnMySourceCode() {
   if (D(CODE(POnMySourceCode))) {
      D_IS_FALSE();
   }
   else {
      D_IS_TRUE();
   }
}
```

Rice's Theorem

- We've now proven that for any property about the "behavior" of programs, D, if…
  - There is some program `D_IS_FALSE()` for which D is false.
  - There is some program `D_IS_TRUE()` for which D is true.
- Then, D does not exist.

Rice's Theorem

- Does P have a NullPointerException?
- Do P and Q do the same thing?
- Does P output 0 on any input?
- Does P have a buffer overflow?
- Does P have a virus?
- Does P have "dead code"?
- …

~~That's it!~~

BUT WAIT…

THERE'S MORE!

Church-Turing Thesis

"All physically computable functions are Java-decidable"

That is, **there is no programming language more powerful than Java.**