CSE 311

Foundations of Computing I

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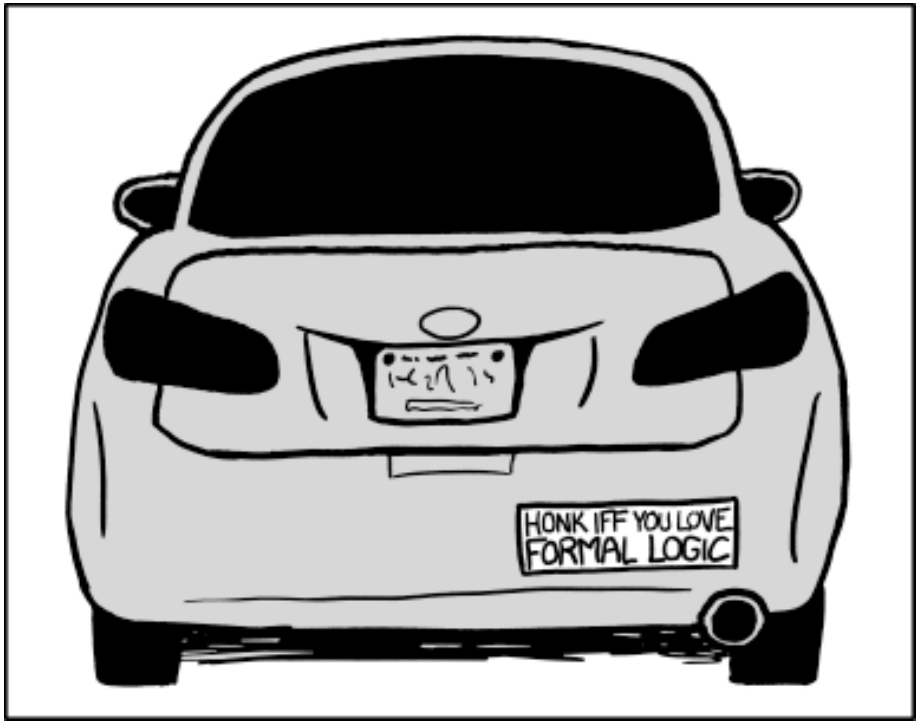
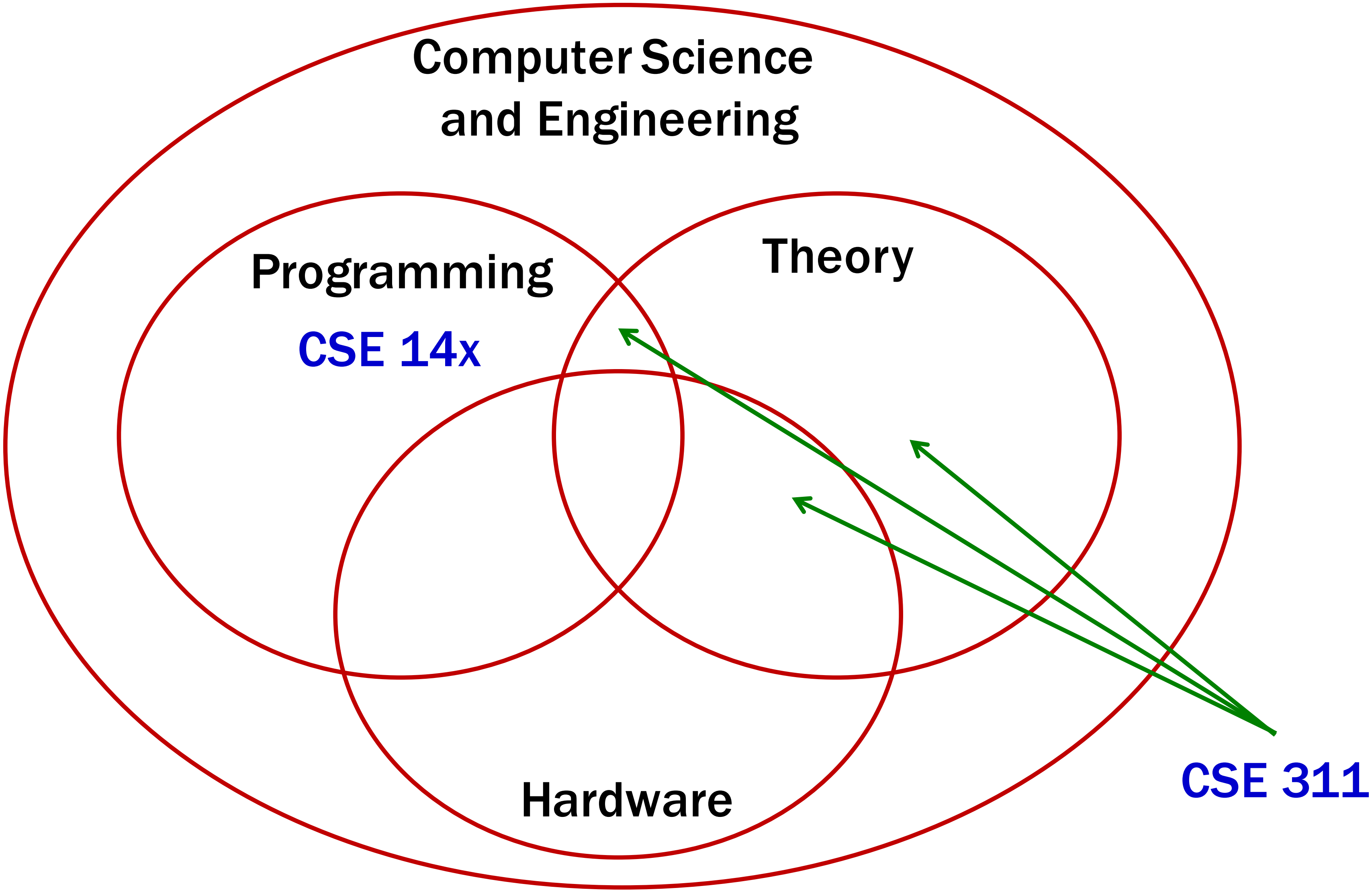
Lecture 1: Propositional Logic



Some Perspective



About the Course

We will study the *theory* needed for CSE:

Logic:

How can we describe ideas *precisely*?

Formal Proofs:

How can we be *positive* we're correct?

Number Theory:

How do we keep data *secure?*

Relations/Relational Algebra:

How do we store information?

Finite State Machines:

How do we design hardware and software?

Turing Machines:

Are there problems computers *can't* solve?

About the Course

It's about perspective!

- Example: Sudoku
  - Given *one*, solve it by hand
  - Given *most*, solve them with a program
  - Given *any*, solve it with computer science

- Tools for reasoning about difficult problems
- Tools for communicating ideas, methods, objectives...
- Tools for automating difficult problems
- Fundamental structures for computer science

This is NOT a programming course!

Administrivia

Instructor: Adam Blank

Teaching Assistants:

- John Armstrong
- Phillip Huang
- Johan Michalove
- Emerson Matson
- Melissa Medsker
- Logan Weber
- Jefferson Van Wagenen
- Ollin Boer Bohan
- Jasper Hugunin
- Michael Lee
- Evan McCarty
- Matthew Rockett

(Optional) Books:

Rosen, Velleman, MIT Book

Don't buy new copies!

Homework:

Due WED at start of class

Write up individually

Section:

Thursdays

Grading (roughly):

50% Homework

20% Midterm

30% Final Exam

All Course Information @ cs.uw.edu/311

Logic: The Language of Reasoning

Why not use English?

- Turn right here…

  Does "right" mean the direction or now?

- Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo

  This means "Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.

- We saw her duck

  Does "duck" mean the animal or crouch down?

"Language of Reasoning" like Java or English

- Words, sentences, paragraphs, arguments…
- Today is about **words** and **sentences**

Why Learn A New Language?

Logic, as the “language of reasoning”, will help us…

- Be more **precise**
- Be more **concise**
- Figure out what a statement means more **quickly**

Propositions

A **proposition** is a statement that

– has a truth value, and

– is “well-formed”

Are These Propositions?

2 + 2 = 5

This is a proposition. It's okay for propositions to be false.

The home page renders correctly in IE.

This is a proposition. It's okay for propositions to be false.

Turn in your homework on Wednesday.

This is a "command" which means it doesn't have a truth value.

This statement is false.

This statement does not have a truth value! (If it's true, it's false, and vice versa.)

Akjsdf!

This is not a proposition because it's gibberish.

Who are you?

This is a question which means it doesn't have a truth value.

Every positive even integer can be written as the sum of two primes.

This is a proposition. We don't know if it's true or false, but we know it's one of them!

Propositions

A **proposition** is a statement that

- has a truth value, and
- is "well-formed"

We need a way of talking about *arbitrary* ideas…

Propositional Variables: $p, q, r, s, \ldots$

Truth Values:

- **T** for **true**
- **F** for **false**

A Proposition

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

We'd like to *understand* what this proposition means.

This is where logic comes in. There are pieces that appear multiple times in the phrase (e.g., "Roger has tusks").

These are called atomic propositions. Let's list them:

RElephant： **"Roger is an orange elephant"**

RTusks： **"Roger has tusks"**

RToenails： **"Roger has toenails"**

Putting Them Together

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

RElephant： **"Roger is an orange elephant"**

RTusks： **"Roger has tusks"**

RToenails： **"Roger has toenails"**

Now, we put these together to make the sentence:

RElephant **and** (RToenails **if** RTusks) **and** (RToenails **or** RTusks **or** (RToenails **and** RTusks))

This is the general idea, but now, let's define our *formal language.*

Logical Connectives

Negation (not)	$\neg p$
Conjunction (and)	$p \land q$
Disjunction (or)	$p \lor q$
Exclusive Or	$p \oplus q$
Implication	$p \longrightarrow q$
Biconditional	$p \longleftrightarrow q$

RElephant:

"Roger is an orange elephant"

RTusks:

"Roger has tusks"

RToenails:

"Roger has toenails"

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

RElephant **and** (RToenails **if** RTusks) **and** (RToenails **or** RTusks **or** (RToenails **and** RTusks))

RElephant ∧ (RToenails **if** RTusks) ∧ (RToenails ∨ RTusks ∨ (RToenails ∧ RTusks))

Some Truth Tables

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication

"If it's raining, then I have my umbrella"

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

It's useful to think of implications as promises. That is "Did I lie?"

	It's raining	It's not raining
I have my umbrella	No	No
I do not have my umbrella	**Yes**	No

*The only **lie** is when:*

(a) It's raining AND

(b) I don't have my umbrella

Implication

"If it's raining, then I have my umbrella"

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Are these true?

$$2 + 2 = 4 \to \text{earth is a planet}$$

The fact that these are unrelated doesn't make the statement false! "2 + 2 = 4" is true; "earth is a planet" is true. T→T is true. So, the statement is true.

$$2 + 2 = 5 \to 26 \text{ is prime}$$

Again, these statements may or may not be related. "2 + 2 = 5" is false; so, the implication is true. (Whether 26 is prime or not is irrelevant).

Implication is not a causal relationship!

$p \rightarrow q$

"I have collected all 151 Pokémon only if I am a Pokémon master"

First, let's figure out what this actually means:

"Only a Pokémon master can collect all 151 Pokémon."

To re-phrase as if p, then q, we think about how this statement could be a lie:

(a) I am a Pokémon master

(b) I don't have all 151 Pokémon

So, the implication is

If I have collected all 151 Pokémon, then I am a Pokémon master.

$p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Implication:

- p implies q
- whenever p is true q must be true
- if p then q
- q if p
- p is sufficient for q
- p only if q

Converse, Contrapositive, Inverse

Implication:

$p \rightarrow q$

Contrapositive:

$\neg q \rightarrow \neg p$

Converse:

$q \rightarrow p$

Inverse:

$\neg p \rightarrow \neg q$

How do these relate to each other?

Consider

p: ***x*** **is divisible by 2**

q*:** ***x **is divisible by 4**

	Divisible By 2	Not Divisible By 2
Divisible By 4	4	Nothing Here!
Not Divisible By 4	2	3

Converse, Contrapositive, Inverse

Implication:

$p \rightarrow q$

Converse:

$q \rightarrow p$

Contrapositive:

$\neg q \rightarrow \neg p$

Inverse:

$\neg p \rightarrow \neg q$

Consider

p: *x* is divisible by 2

q: *x* is divisible by 4

$p \rightarrow q$	F
$q \rightarrow p$	T
$\neg q \rightarrow \neg p$	F
$\neg p \rightarrow \neg q$	T

	Divisible By 2	Not Divisible By 2
Divisible By 4	4	Nothing Here!
Not Divisible By 4	2	3

An implication and it's contrapositive have the same truth value!

Back to Roger's Sentence

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

↓

RElephant ∧ (RToenails **if** RTusks) ∧ (RToenails ∨ RTusks ∨ (RToenails ∧ RTusks))

Define shorthand …

p : RElephant
q : RTusks
r : RToenails

↓

$$(p \wedge (q \rightarrow r) \wedge (r \vee q \vee (r \wedge q))$$

Roger's Sentence with a Truth Table

p	q	r	$q \longrightarrow r$	$p \wedge (q \longrightarrow r)$	$r \vee q$	$r \wedge q$	$(r \vee q) \vee (r \wedge q)$	$p \wedge (q \longrightarrow r) \wedge (r \vee q) \vee (r \wedge q)$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T	F
T	F	T	T	T	T	F	T	T
T	F	F	T	T	F	F	F	F
F	T	T	T	F	T	T	T	F
F	T	F	F	F	T	F	T	F
F	F	T	T	F	T	F	T	F
F	F	F	T	F	F	F	F	F

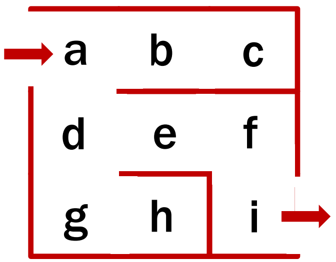
Biconditional: $p \leftrightarrow q$

- *p* iff *q*
- *p* is equivalent to *q*
- *p* implies *q* and *q* implies *p*

p	q	$p \leftrightarrow q$
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

Solving Mazes

"Every cell in the solution of a maze has *two* cells next to it in the solution. Also, "a" is in the solution. Also, "i" is in the solution."



a: **"The cell a is in the solution"**

b: **"The cell b is in the solution"**

....

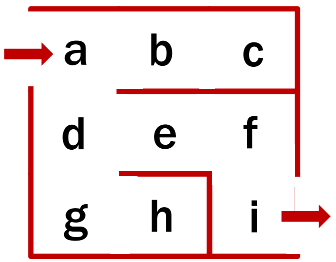
i: **"The cell i is in the solution"**

$$
\begin{aligned}
a \wedge i \wedge (a \longrightarrow (b \wedge d)) \wedge \\
(b \longrightarrow (a \wedge c)) \wedge \\
(c \longrightarrow F) \wedge \\
(d \longrightarrow (???)) \wedge \\
(e \longrightarrow (d \wedge f)) \wedge \\
(f \longrightarrow (e \wedge i)) \wedge \\
(g \longrightarrow (d \wedge h)) \wedge \\
(h \longrightarrow F) \wedge \\
(i \longrightarrow F)
\end{aligned}
$$

Let's fill in the "???"

Solving Mazes

"Every cell in the solution of a maze has *two* cells next to it in the solution. Also, "a" is in the solution. Also, "i" is in the solution."



a: **"The cell a is in the solution"**

b: **"The cell b is in the solution"**

....

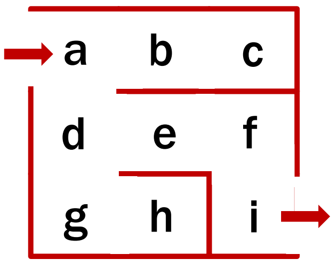
i: **"The cell i is in the solution"**

$$
\begin{aligned}
a \wedge i \wedge (a \longrightarrow (b \wedge d)) \wedge \\
(b \longrightarrow (a \wedge c)) \wedge \\
(c \longrightarrow F) \wedge \\
(d \longrightarrow ((a \wedge e) \vee (a \wedge g) \vee (e \wedge g))) \wedge \\
(e \longrightarrow (d \wedge f)) \wedge \\
(f \longrightarrow (e \wedge i)) \wedge \\
(g \longrightarrow (d \wedge h)) \wedge \\
(h \longrightarrow F) \wedge \\
(i \longrightarrow F)
\end{aligned}
$$

There's something wrong here. Look at i.

Solving Mazes

"Every cell in the solution of a maze has *two* cells next to it in the solution except "a"/"i" which need one. Also, "a" is in the solution. Also, "i" is in the solution."



a: "The cell a is in the solution"

b: "The cell b is in the solution"

....

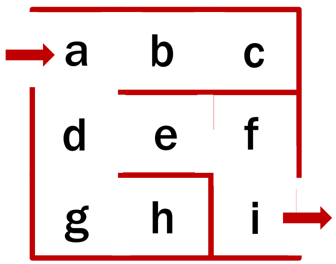
i: "The cell i is in the solution"

$$a \wedge i \wedge (a \longrightarrow (b \wedge d)) \wedge$$
$$(b \longrightarrow (a \wedge c)) \wedge$$
$$(c \longrightarrow F) \wedge$$
$$(d \longrightarrow ((a \wedge e) \vee (a \wedge g) \vee (e \wedge g))) \wedge$$
$$(e \longrightarrow (d \wedge f)) \wedge$$
$$(f \longrightarrow (e \wedge i)) \wedge$$
$$(g \longrightarrow (d \wedge h)) \wedge$$
$$(h \longrightarrow F) \wedge$$
$$(i \longrightarrow F)$$

There's something wrong here. Look at i.

Solving Mazes

$$
\begin{aligned}
a \wedge i \wedge (a &\longrightarrow (\mathbf{b} \vee \mathbf{d})) \wedge \\
&(b \longrightarrow (a \wedge c)) \wedge \\
&(c \longrightarrow F) \wedge \\
&(d \longrightarrow ((a \wedge e) \vee (a \wedge g) \vee (e \wedge g))) \wedge \\
&(e \longrightarrow (d \wedge f)) \wedge \\
&(f \longrightarrow (e \wedge i)) \wedge \\
&(g \longrightarrow (d \wedge h)) \wedge \\
&(h \longrightarrow F) \wedge \\
&(i \longrightarrow \mathbf{f})
\end{aligned}
$$



Truth Table Time?

Let's not…this looks horrible!

This is what computers are for.