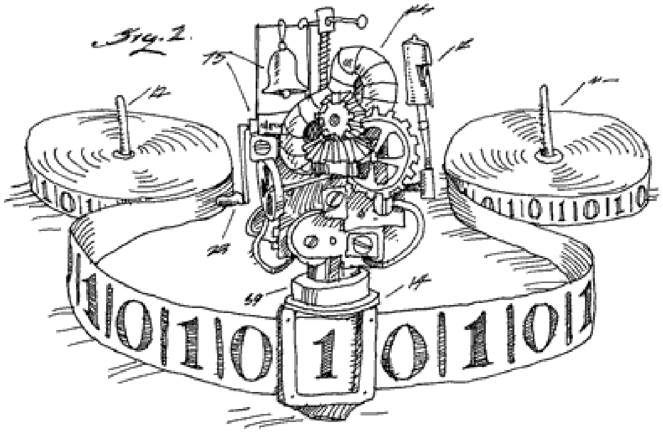
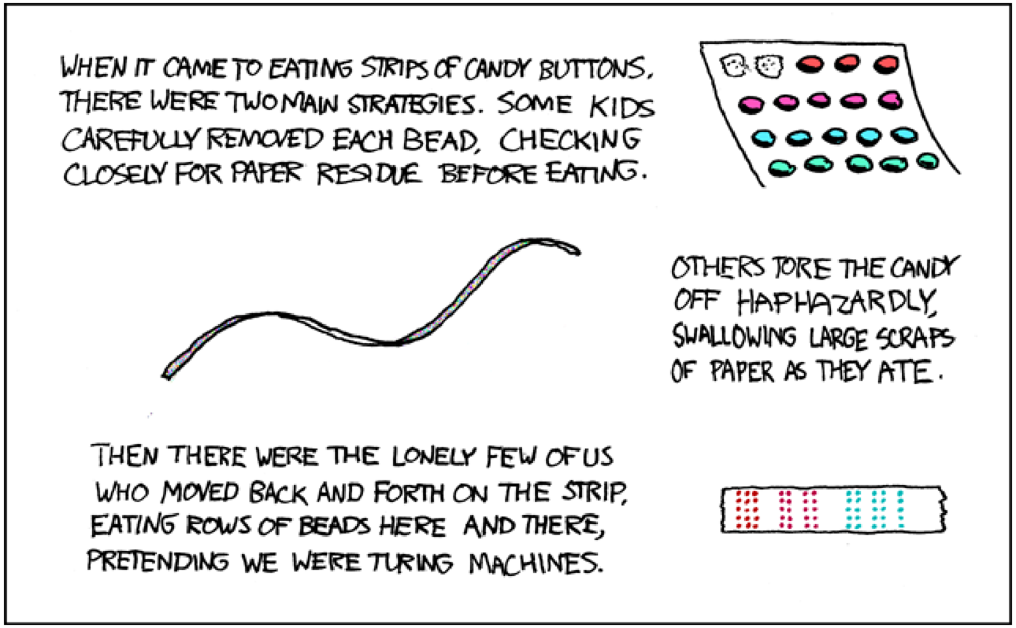
CSE 311: Foundations of Computing

Lecture 29: Turing Machines




WHEN IT CAME TO EATING STRIPS OF CANDY BUTTONS, THERE WERE TWO MAIN STRATEGIES. SOME KIDS CAREFULLY REMOVED EACH BEAD, CHECKING CLOSELY FOR PAPER RESIDUE BEFORE EATING.
OTHERS TORE THE CANDY OFF HAPHAZARDLY, SWALLOWING LARGE SCRAPS OF PAPER AS THEY ATE.
THEN THERE WERE THE LONELY FEW OF US WHO MOVED BACK AND FORTH ON THE STRIP, EATING ROWS OF BEADS HERE AND THERE, PRETENDING WE WERE TURING MACHINES.

Final exam

- **Monday** at either **2:30-4:20 p.m.** or **4:30-6:20 p.m.**
  - **Kane Hall 220**
  - You need to fill out **Catalyst Survey** to say which you are taking by **midnight Sunday** night.
  - Bring your **UW ID** and have it out and ready during the exam
- **Comprehensive** coverage. If you had a homework question on it, it is fair game. See link on webpage.
  - Includes pre-midterm topics, e.g. formal proofs. Will contain the same sheets at end.
- **Review session: Sunday 4:00-6:00 p.m. EEB 105**
  - **Bring your questions !!**

Homework

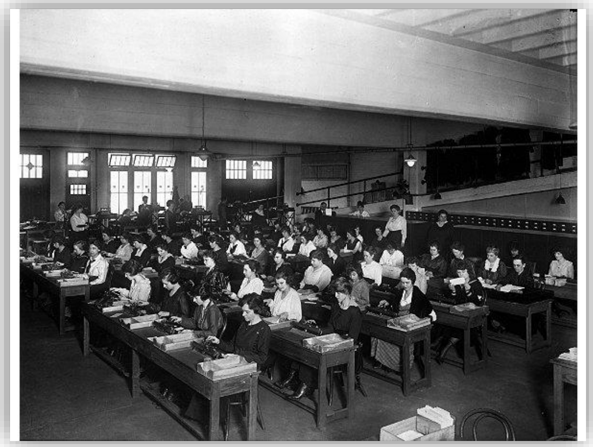
- Homework 9
  - Due today at 5:00 p.m.
  - Grading won't be finished until after the final

- Look for `cse311` e-mail with pointer to solutions for Homework 9 overnight tonight
  - Will be on password-protected webpage

Computers and algorithms

- Does Java (or any programming language) cover all possible computation? Every possible algorithm?

- There was a time when computers were people who did calculations on sheets paper to solve computational problems



- Computers as we known them arose from trying to understand everything these people could do.

Before Java

1930's:

How can we formalize what algorithms are possible?

- **Turing machines** (Turing, Post)
  - basis of modern computers
- **Lambda Calculus** (Church)
  - basis for functional programming, LISP
- **μ-recursive functions** (Kleene)
  - alternative functional programming basis

Turing machines

Church-Turing Thesis:

Any reasonable model of computation that includes all possible algorithms is equivalent in power to a Turing machine

Evidence

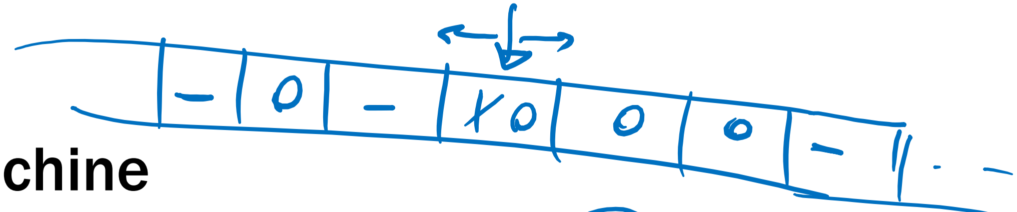
- Intuitive justification
- Huge numbers of models based on radically different ideas turned out to be equivalent to TMs

Turing machines

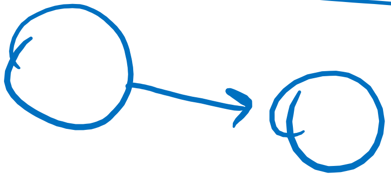
- **Finite Control**
  - **Brain/CPU** that has only a **finite # of possible "states of mind"**
- **Recording medium**
  - **An unlimited supply of** blank "**scratch paper**" on which to **write & read symbols,** each chosen from a **finite set of possibilities**
  - **Input** also supplied **on** the **scratch paper**
- **Focus of attention**
  - **Finite control** can only **focus on** a **small portion of the recording medium at once**
  - **Focus of attention** can only **shift a small amount at a time**

Turing machines

- **Recording medium**
  - An **infinite** read/write "**tape**" marked off into **cells**
  - Each **cell** can store **one symbol or** be "**blank**"
  - **Tape** is **initially** all **blank except** a few **cells** of the tape **containing the input string**
  - **Read/write head** can **scan one cell** of the tape - **starts on input**



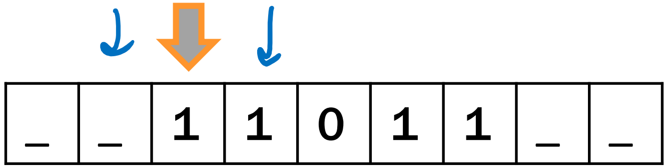
- **In each step,** **a Turing machine**
  1. **Reads** the **currently scanned cell**
  2. **Based on current state and scanned symbol**
     1. **Overwrites symbol in scanned cell**
     2. **Moves read/write head left or right one cell**
     3. **Changes to a new state**



- **Each Turing Machine is specified by its finite set of rules**

Turing machines

	_	0	1
s_1	(1, L, s_3)	(1, L, s_4)	(0, R, s_2)
s_2	(0, R, s_1)	(1, R, s_1)	(0, R, s_1)
s_3			
s_4			



_	_	1	1	0	1	1	_	_

UW CSE’s Steam-Powered Turing Machine

ACCEPT
REJECT

Original in Sieg Hall stairwell

Turing machines

Ideal Java/C programs:

- Just like the Java/C you're used to programming with, except you never run out of memory
  - Constructor methods always succeed
  - **malloc** in C never fails

Equivalent to Turing machines except a lot easier to program:

- Turing machine definition is useful for breaking computation down into simplest steps
- We only care about high level so we use programs

Turing's big idea part 1: Machines as data

Original Turing machine definition:

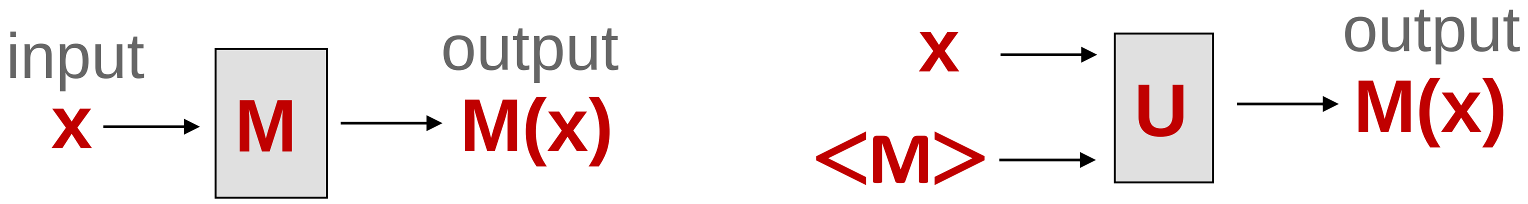
- A different "machine" **M** for each task
- Each machine **M** is defined by a finite set of possible operations on finite set of symbols
- So... **M** has a finite description as a sequence of symbols, its "code", which we denote **<M>**

You already are used to this idea with the notion of the program code or text but this was a new idea in Turing's time.

Turing’s big idea part 2: A Universal TM

- **A Turing machine interpreter U**
  - On input <M> and its input x,
    U outputs the same thing as M does on input x
  - At each step it decodes which operation M would have performed and simulates it.
- **One Turing machine is enough**
  - Basis for modern stored-program computer

    Von Neumann studied Turing’s UTM design



Takeaway from undecidability

- You can’t rely on the idea of improved compilers and programming languages to eliminate major programming errors
  - truly safe languages can’t possibly do general computation
- Document your code
  - there is no way you can expect someone else to figure out what your program does with just your code; since in general it is provably impossible to do this!

We’ve come a long way!

- Propositional Logic.
- Boolean logic and circuits.
- Boolean algebra.
- Predicates, quantifiers and predicate logic.
- Inference rules and formal proofs for propositional and predicate logic.
- English proofs.
- Set theory.
- Modular arithmetic.
- Prime numbers.
- GCD, Euclid's algorithm, modular inverse, and exponentiation.

We’ve come a long way!

- Induction and Strong Induction.
- Recursively defined functions and sets.
- Structural induction.
- Regular expressions.
- Context-free grammars and languages.
- Relations and composition.
- Transitive-reflexive closure.
- Graph representation of relations and their closures.

We’ve come a long way!

- DFAs, NFAs and language recognition.
- Product construction for DFAs.
- Finite state machines with outputs at states.
- Minimization algorithm for finite state machines
- Conversion of regular expressions to NFAs.
- Subset construction to convert NFAs to DFAs.
- Equivalence of DFAs, NFAs, Regular Expressions
- Finite automata for pattern matching.
- Method to prove languages not accepted by DFAs.
- Cardinality, countability and diagonalization
- Undecidability: Halting problem and evaluating properties of programs.

What's next? ...after the final exam...

- **Foundations II** (3~~2~~1) 12
  - Fundamentals of counting, discrete probability, applications of randomness to computing, statistical algorithms and analysis
  - Ideas critical for machine learning, algorithms

- **Data Abstractions** (332)
  - Data structures, a few key algorithms, parallelism
  - Brings programming and theory together
  - Makes heavy use of induction and recursive defns

Course Evaluation Online

- This should be filled out by Sunday night
  - Your ability to fill it out will disappear at 11:59 p.m. on Sunday.
  - It will be worth your while to fill it out!

- Look for an important message on the cse311 mailing list about course evaluations this evening

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