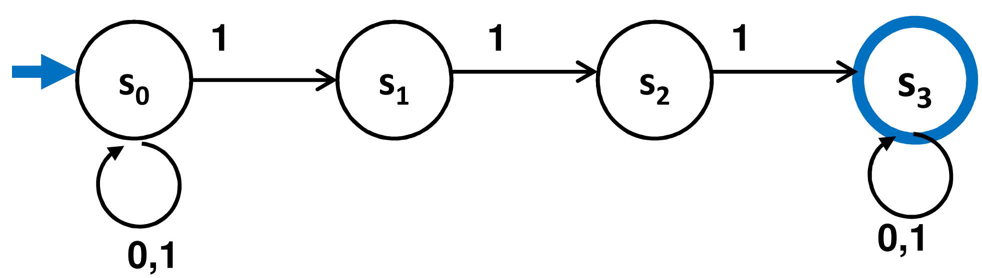
CSE 311: Foundations of Computing

Lecture 24: NFAs, Regular expressions, and NFA→DFA

Nondeterministic Finite Automata (NFA)

- Graph with start state, final states, edges labeled by symbols (like DFA) but
  - Not required to have exactly 1 edge out of each state labeled by each symbol— can have 0 or >1
  - Also can have edges labeled by empty string ε
- **Defn:** x is in the language recognized by an NFA if and only if x labels a path from the start state to some final state



Three ways of thinking about NFAs

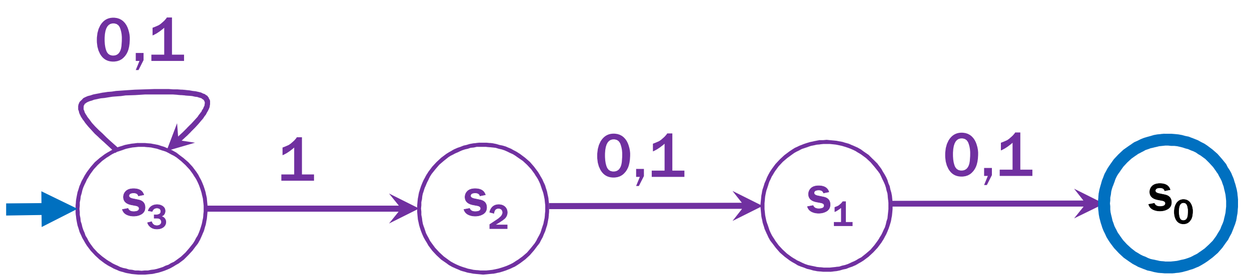
- **Outside observer: Is there a path labeled by** x **from the start state to some final state?**

- **Perfect guesser: The NFA has input** x **and whenever there is a choice of what to do it magically guesses a good one (if one exists)**

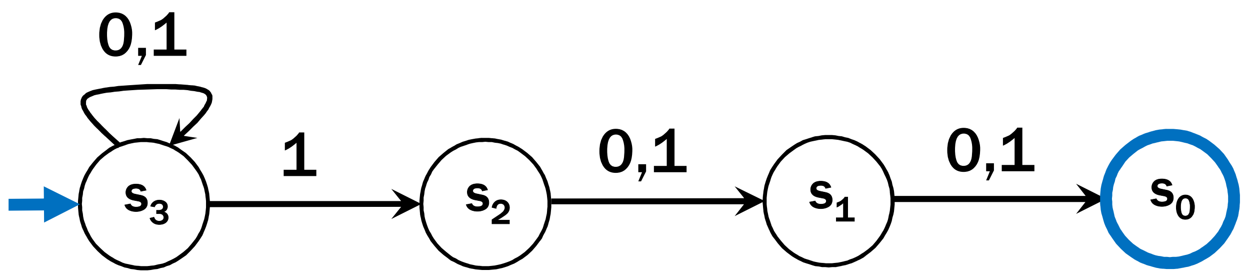
- **Parallel exploration: The NFA computation runs all possible computations on** x **step-by-step at the same time in parallel**

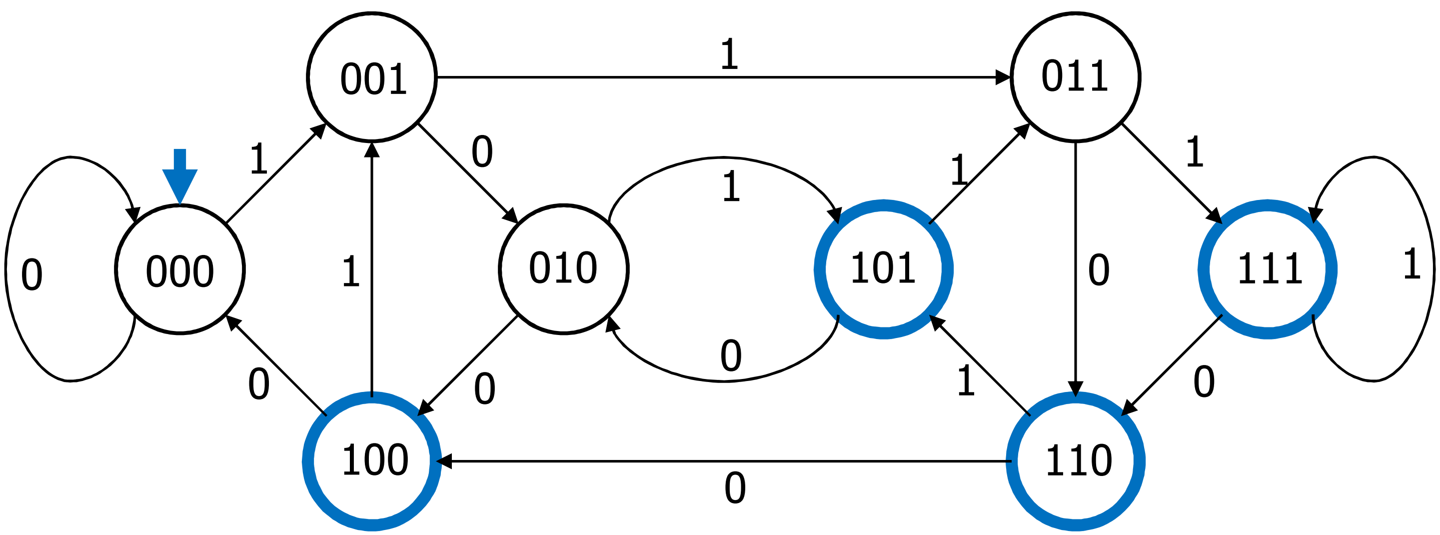
NFA for set of binary strings with a 1 in the 3rd position from the end

NFA for set of binary strings with a 1 in the 3rd position from the end

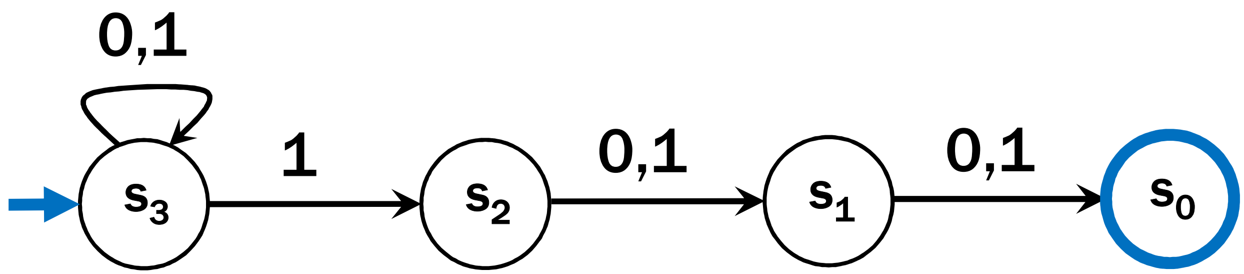


Compare with the DFA

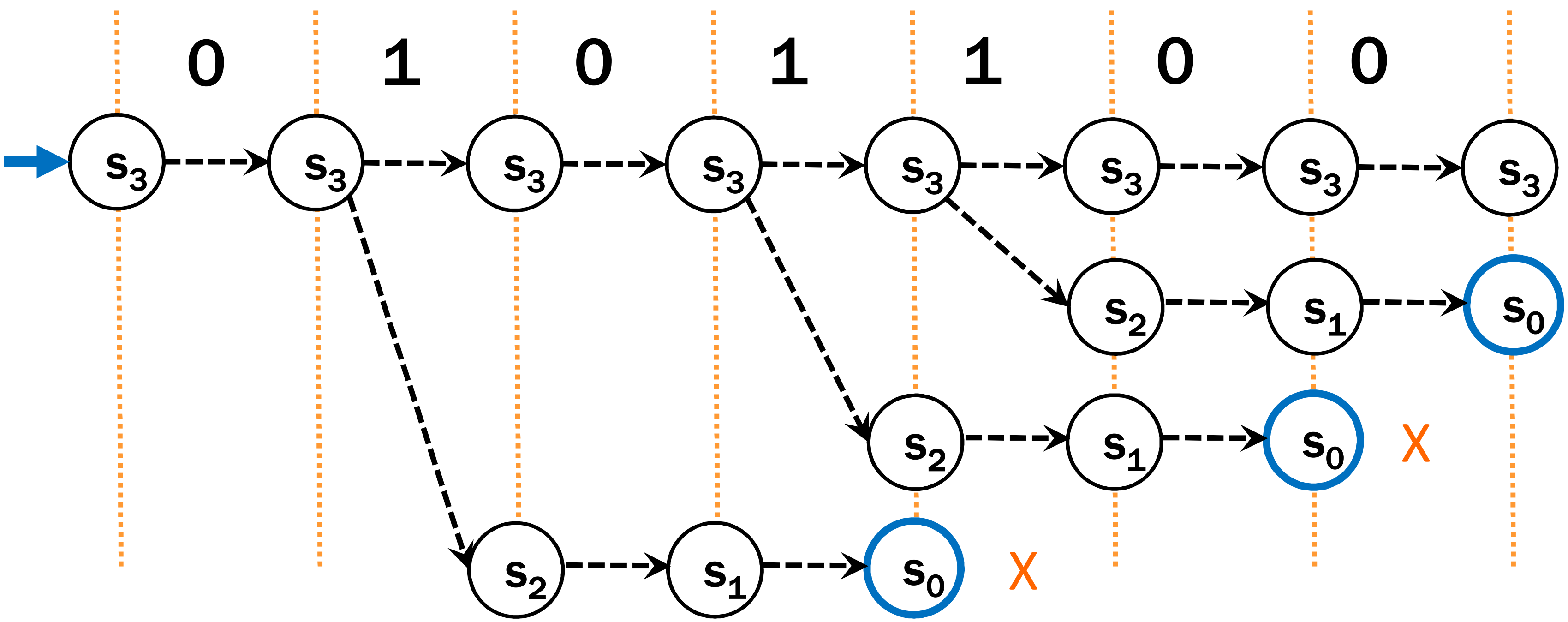




Parallel Exploration view of an NFA



Input string 0101100



NFAs and regular expressions

Theorem: For any set of strings (language) A described by a regular expression, there is an NFA that recognizes A.

Proof idea: Structural induction based on the recursive definition of regular expressions...

Regular Expressions over Σ

- **Basis:**
  - $\varnothing$, ε are regular expressions
  - *a* is a regular expression for any $a \in \Sigma$
- **Recursive step:**
  - If A and B are regular expressions then so are:

    $(A \cup B)$

    (AB)

    A^*

Base Case

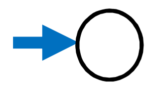
- **Case ∅:**

- **Case ε:**

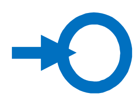
- **Case *a*:**

Base Case

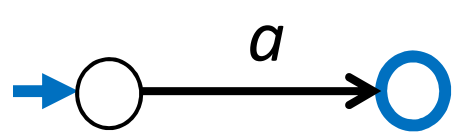
- Case ∅:



- Case ε:

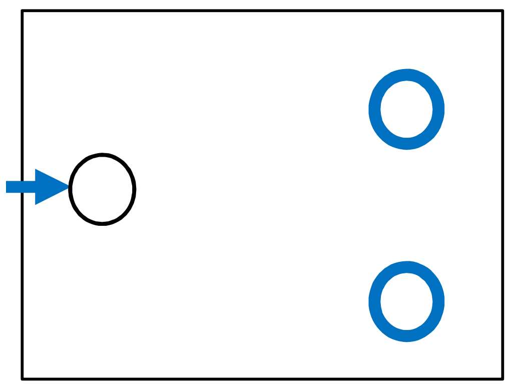


- Case *a*:

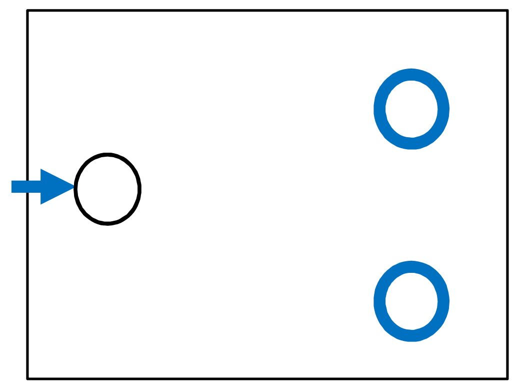


Inductive Hypothesis

- Suppose that for some regular expressions A and B there exist NFAs N_A and N_B such that N_A recognizes the language given by A and N_B recognizes the language given by B



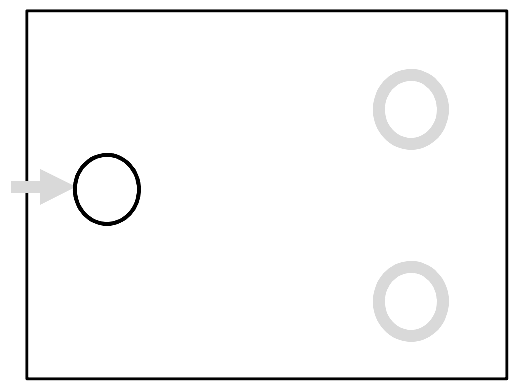
N_A



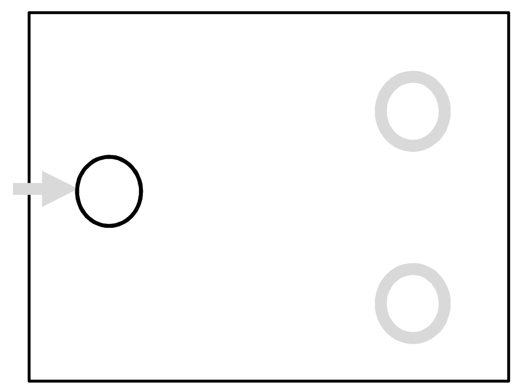
N_B

Inductive Step

Case ($A \cup B$):



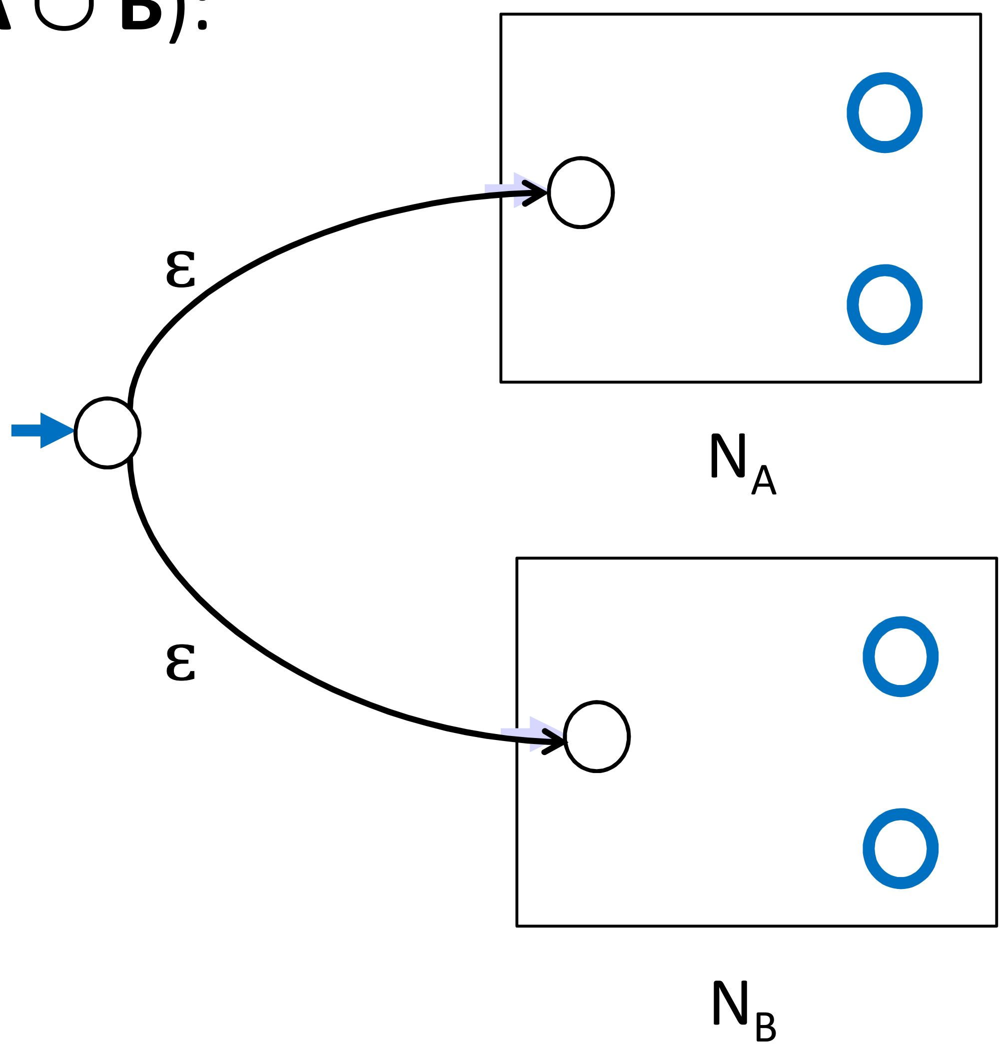
N_A



N_B

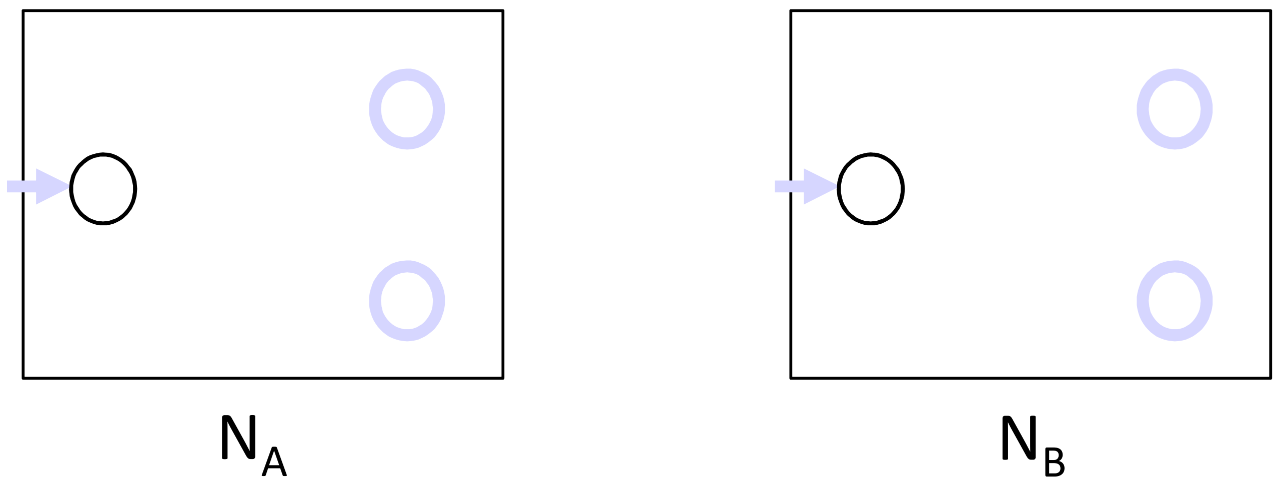
Inductive Step

Case (A ∪ B):



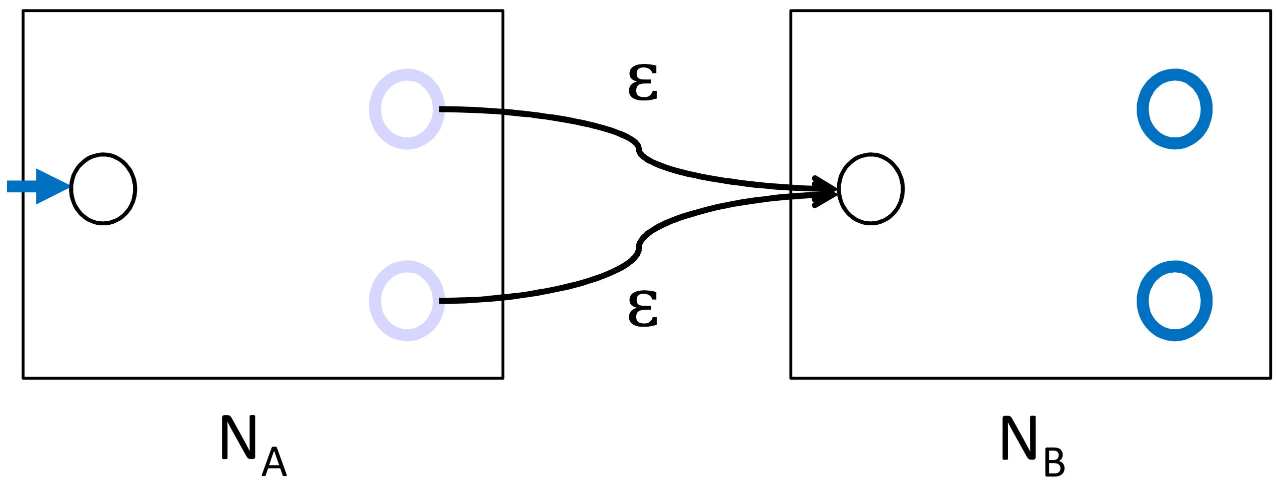
Inductive Step

Case (**AB**):



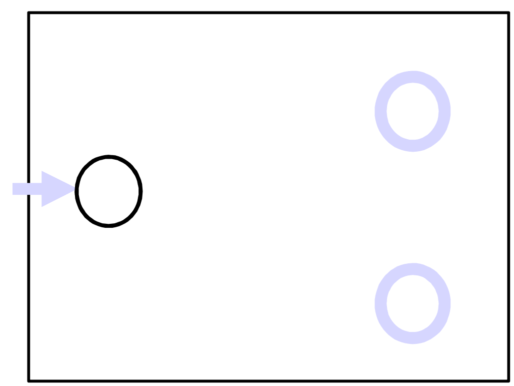
Inductive Step

Case (**AB**):



Inductive Step

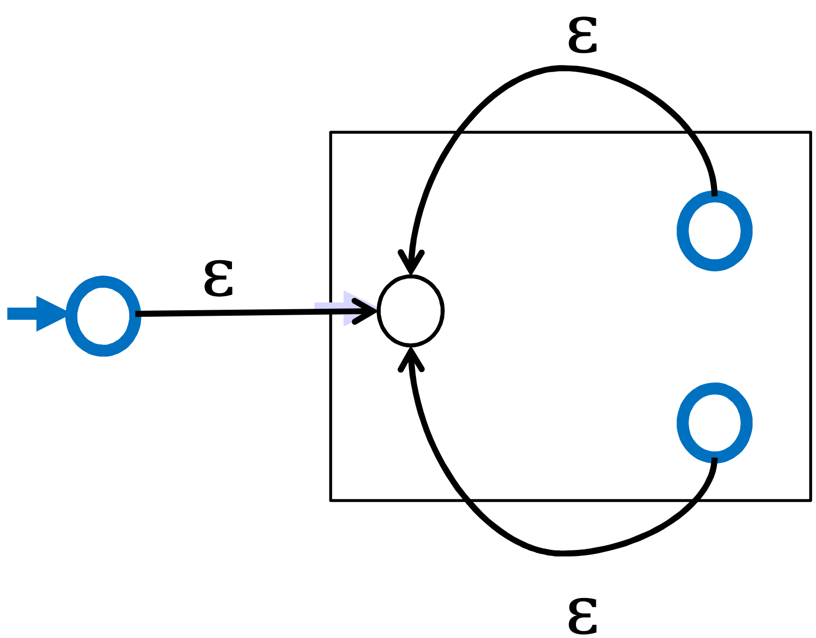
Case A*



N_A

Inductive Step

Case A*

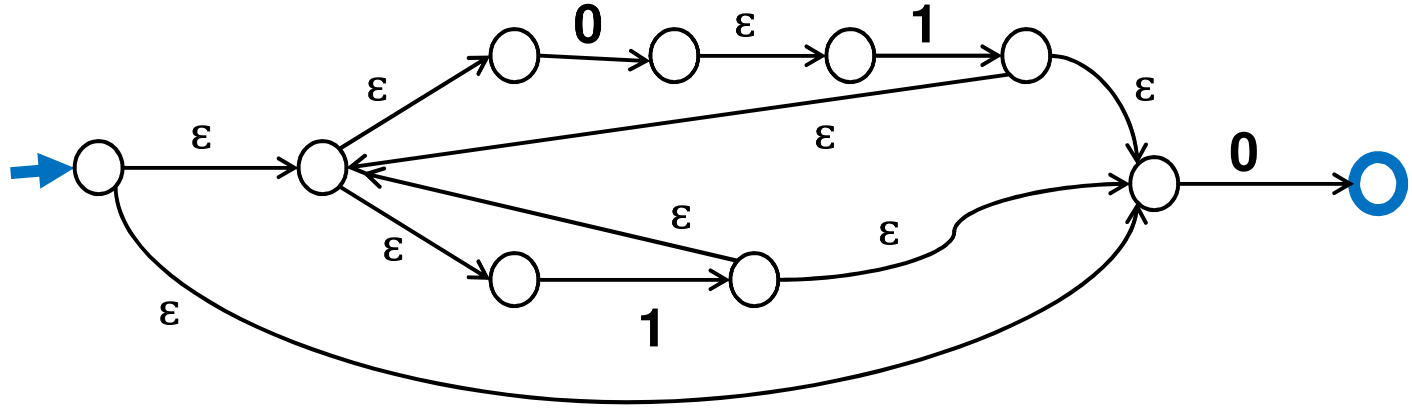


N_A

Build an NFA for $(01 \cup 1)^*0$

Solution

(01 ∪1)*0



NFAs and DFAs

Every DFA is an NFA

– DFAs have requirements that NFAs don't have

Can NFAs recognize more languages?

NFAs and DFAs

Every DFA is an NFA

– DFAs have requirements that NFAs don't have

Can NFAs recognize more languages? No!

Theorem: For every NFA there is a DFA that recognizes exactly the same language

Three ways of thinking about NFAs

- **Outside observer:** Is there a path labeled by x from the start state to some final state?

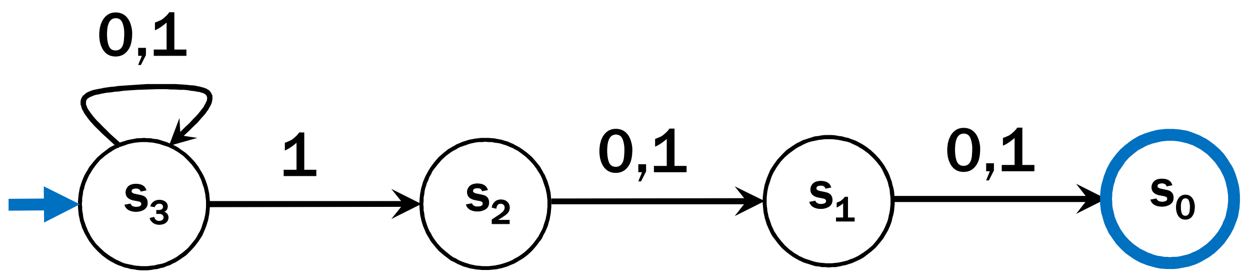
- **Perfect guesser:** The NFA has input x and whenever there is a choice of what to do it magically guesses a good one (if one exists)

- **Parallel exploration:** The NFA computation runs all possible computations on x step-by-step at the same time in parallel

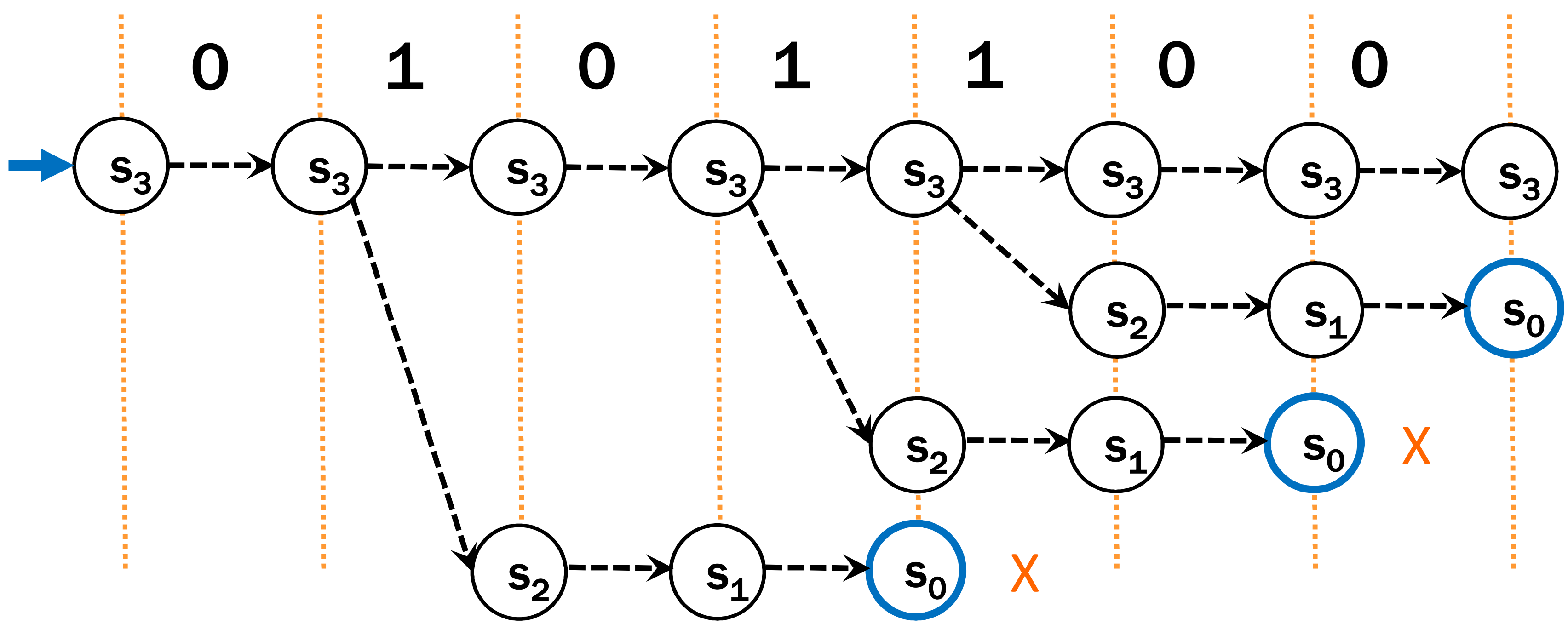
Conversion of NFAs to a DFAs

- Proof Idea:
  - The DFA keeps track of ALL the states that the part of the input string read so far can reach in the NFA
  - There will be one state in the DFA for each *subset* of states of the NFA that can be reached by some string

Parallel Exploration view of an NFA



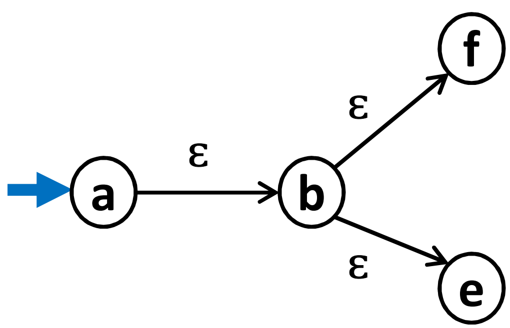
Input string 0101100



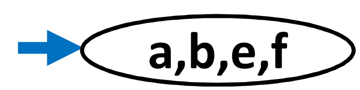
Conversion of NFAs to a DFAs

New start state for DFA

– The set of all states reachable from the start state of the NFA using only edges labeled ε



NFA

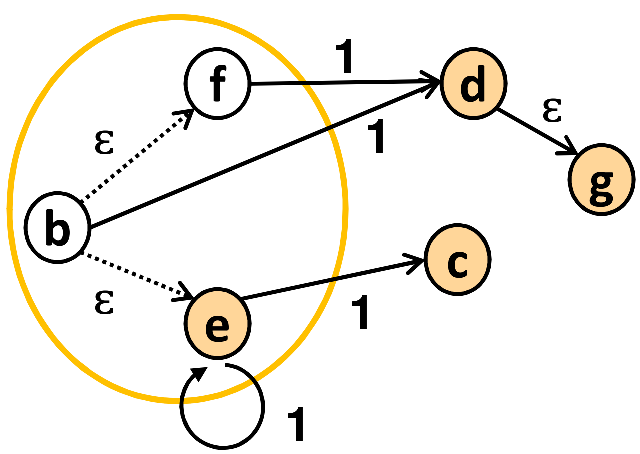


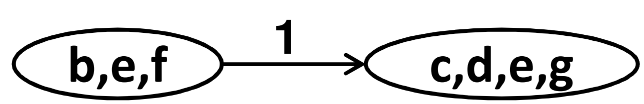
DFA

Conversion of NFAs to a DFAs

For each state of the DFA corresponding to a set S of states of the NFA and each symbol s

- **Add an edge labeled s to state corresponding to T, the set of states of the NFA reached by**
  - starting from some state in S, then
  - following one edge labeled by s, and
    then following some number of edges labeled by ε
- **T will be ∅ if no edges from S labeled s exist**

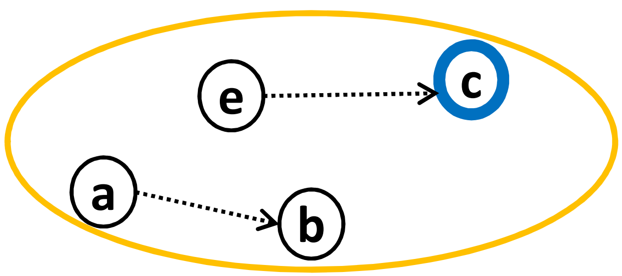




Conversion of NFAs to a DFAs

Final states for the DFA

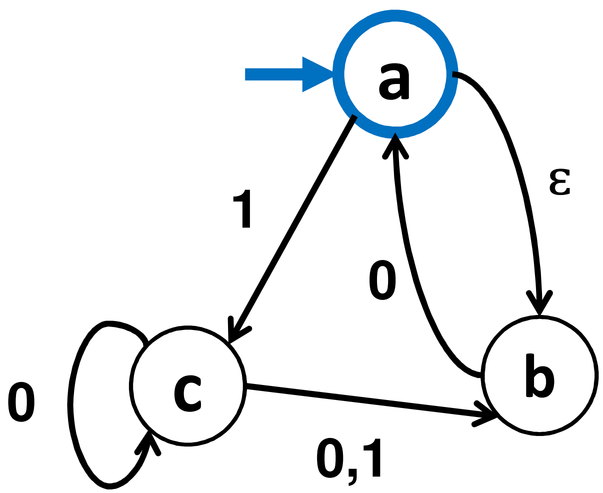
– All states whose set contain some final state of the NFA



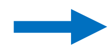
NFA

DFA

Example: NFA to DFA

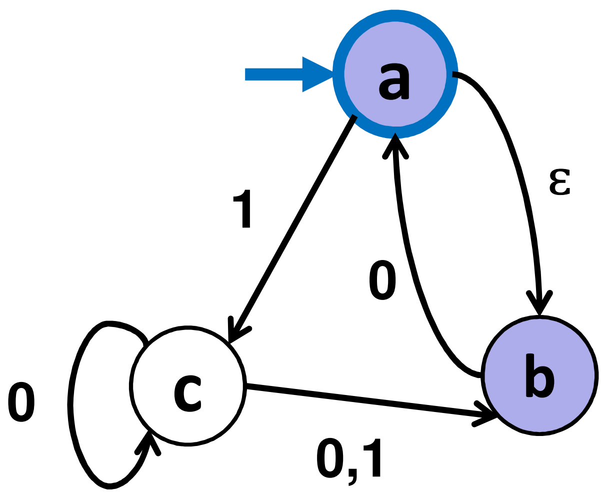


NFA

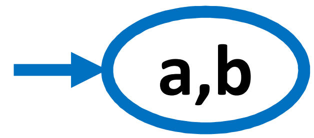


DFA

Example: NFA to DFA

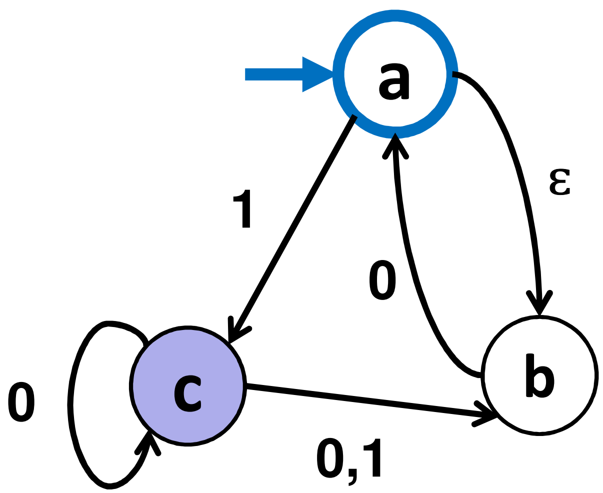


NFA

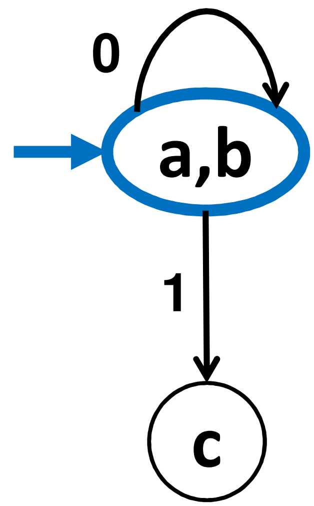


DFA

Example: NFA to DFA

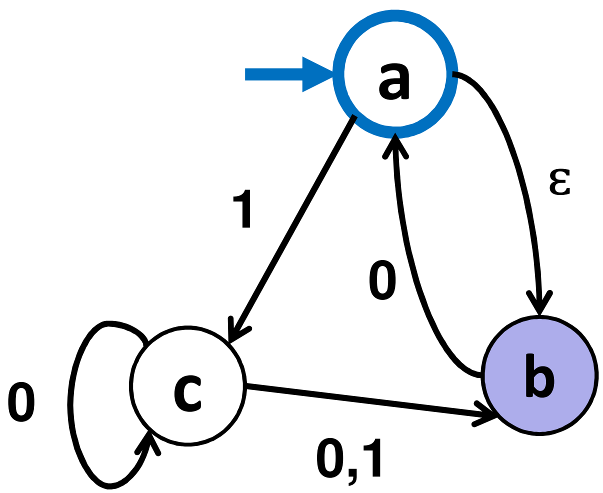


NFA

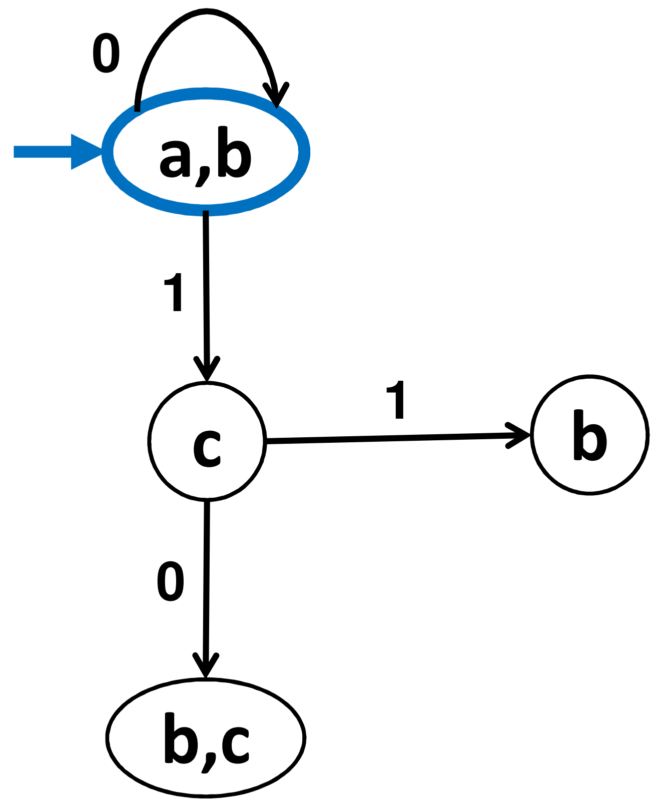


DFA

Example: NFA to DFA

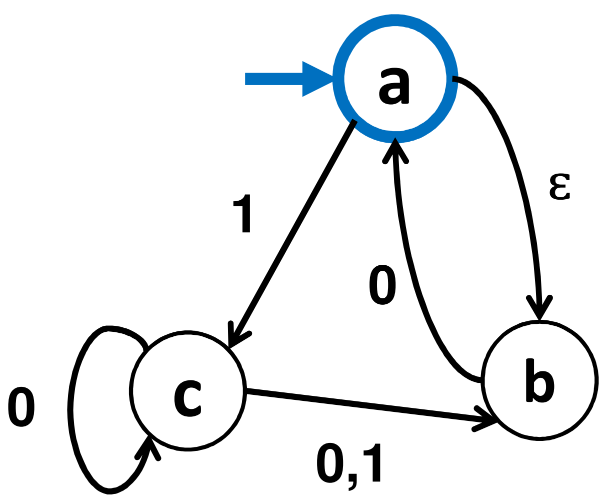


NFA

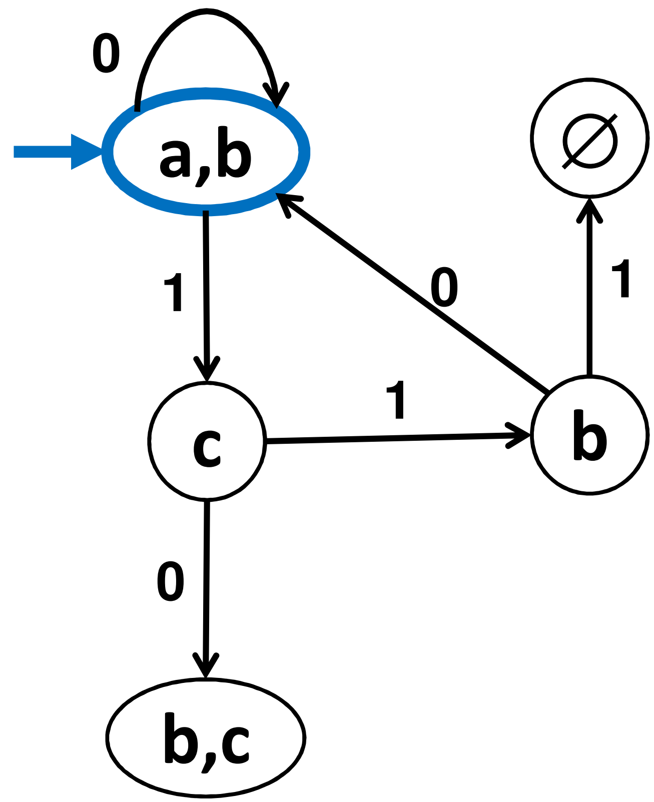


DFA

Example: NFA to DFA

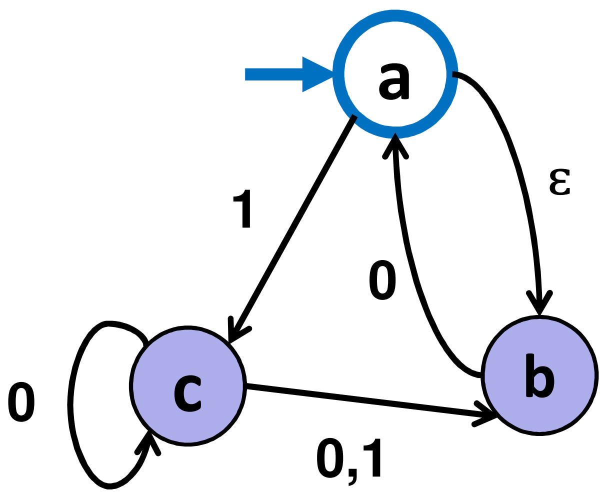


NFA

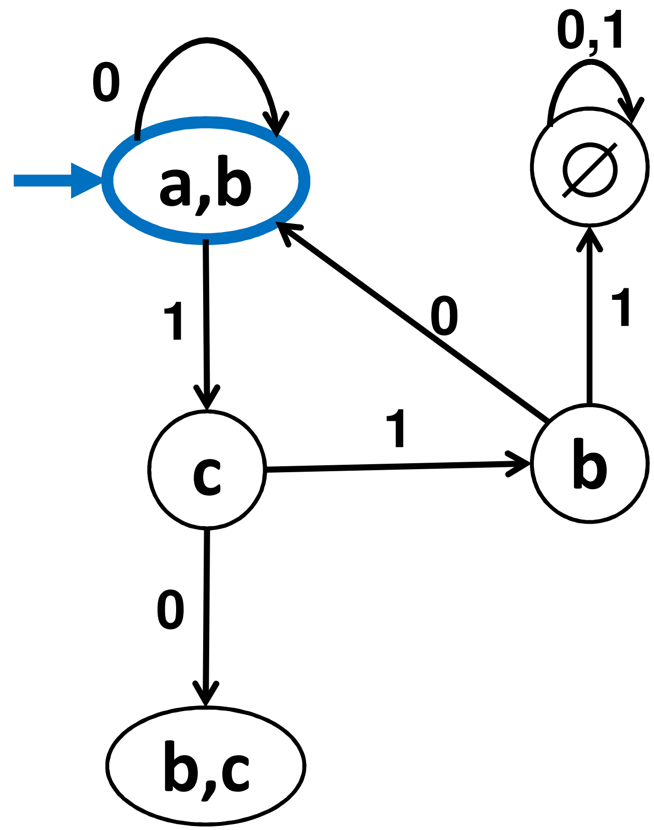


DFA

Example: NFA to DFA

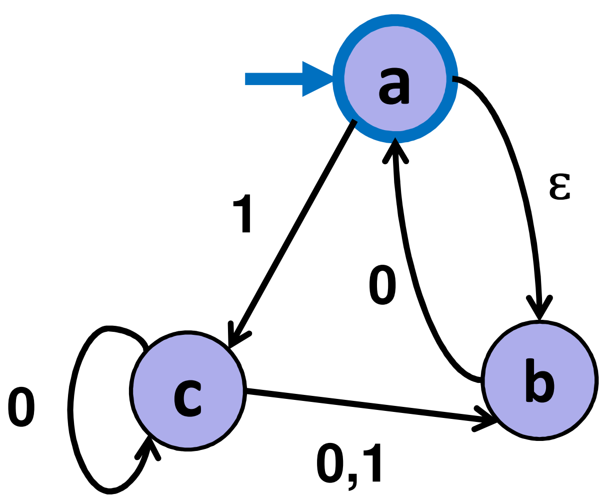


NFA

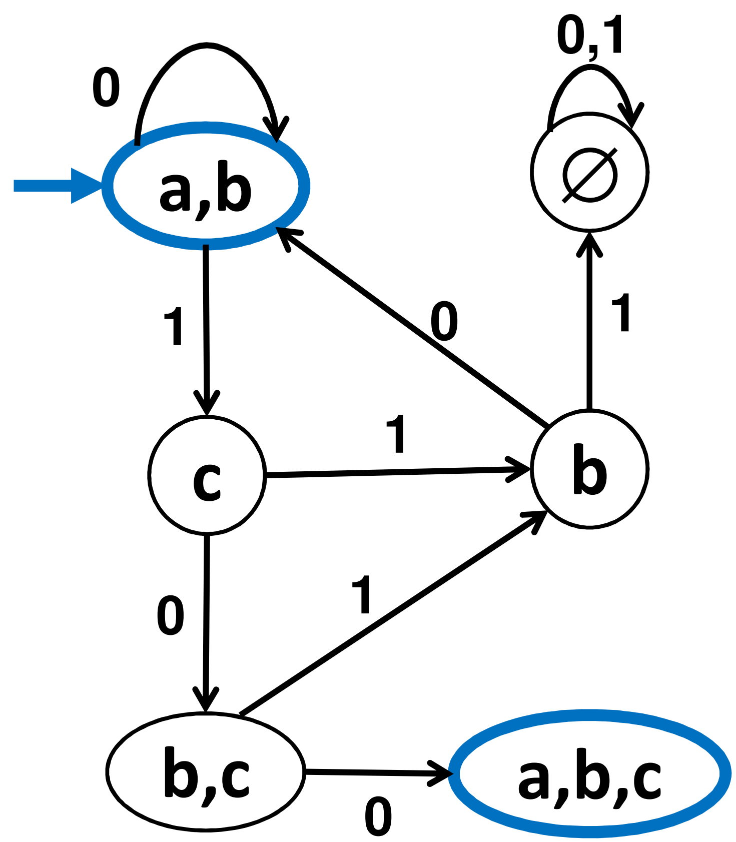


DFA

Example: NFA to DFA

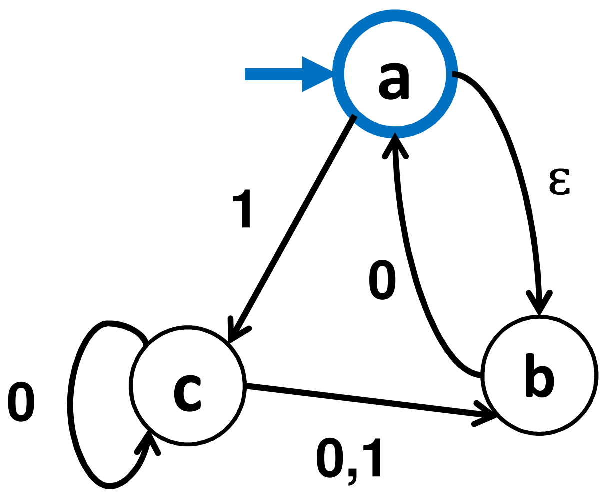


NFA

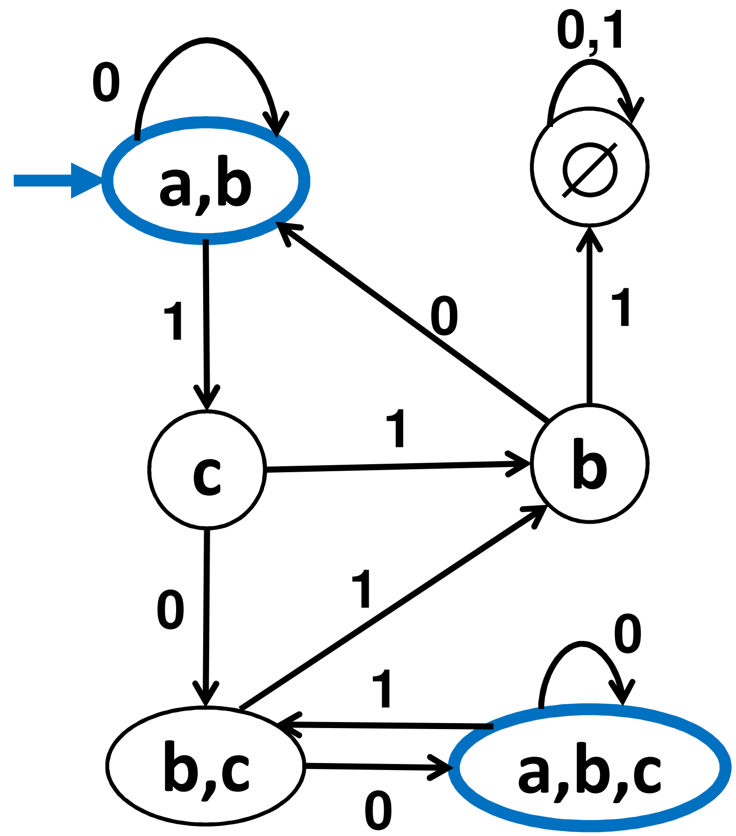


DFA

Example: NFA to DFA



NFA



DFA

Exponential Blow-up in Simulating Nondeterminism

- **In general the DFA might need a state for every subset of states of the NFA**
  - Power set of the set of states of the NFA
  - n-state NFA yields DFA with at most 2^n states
  - We saw an example where roughly 2^n is necessary
    "Is the n^{th} char from the end a 1?"

- **The famous "P=NP?" question asks whether a similar blow-up is always necessary to get rid of nondeterminism for polynomial-time algorithms**