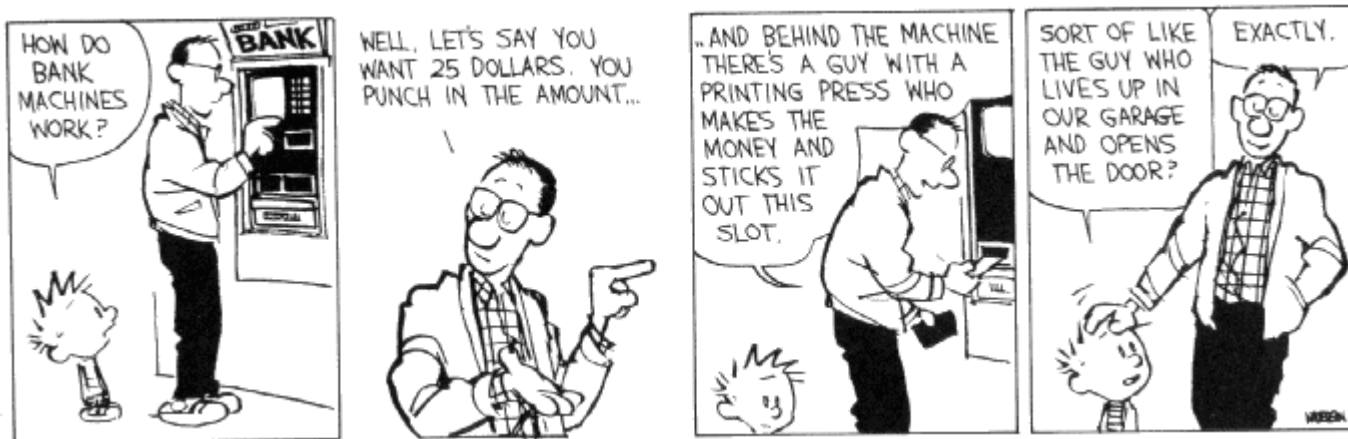


CSE 311: Foundations of Computing

Lecture 22: DFAs and Finite State Machines with Output

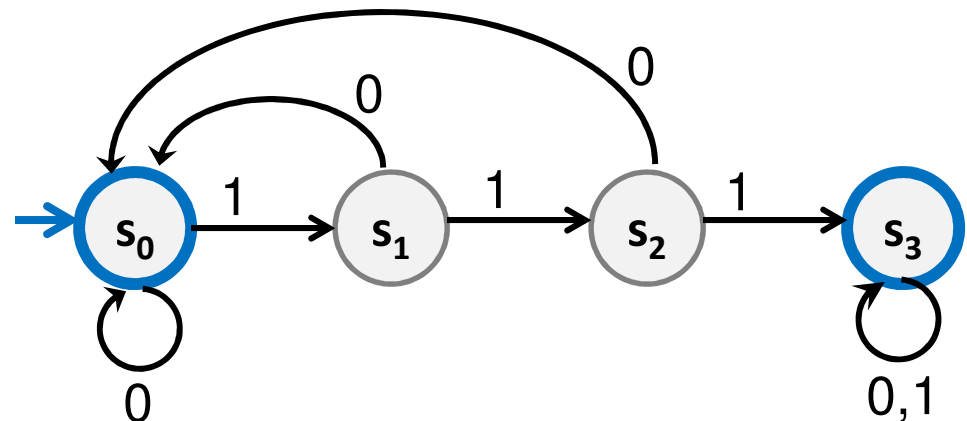


Finite State Machines

DFA's

- States
- Transitions on input symbols
- Start state and final states
- The “language recognized” by the machine is the set of strings that reach a final state from the start

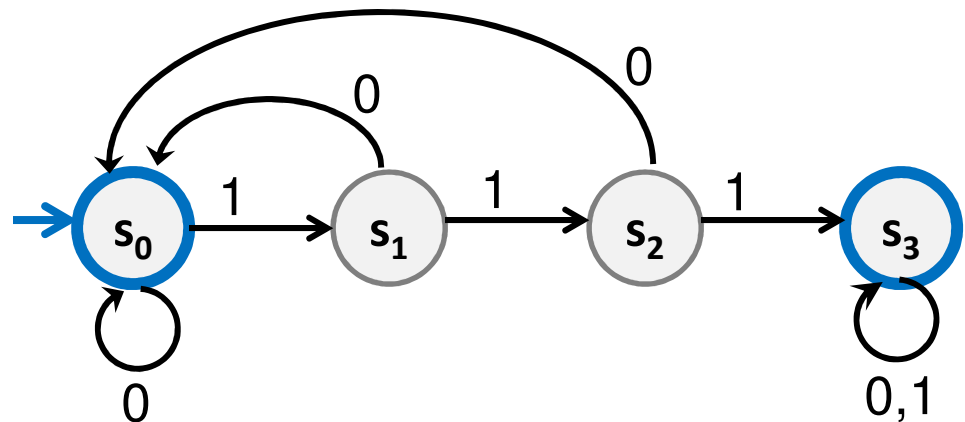
Old State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3



Finite State Machines

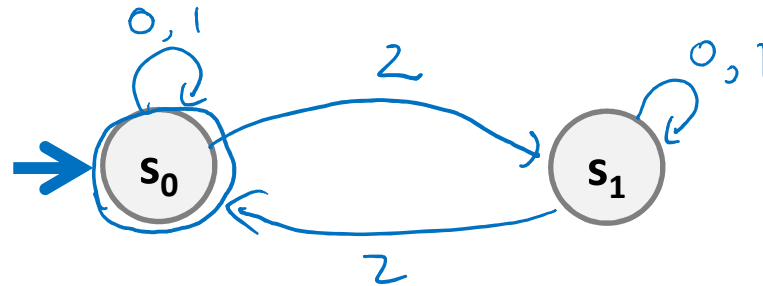
- Each machine designed for strings over some fixed alphabet Σ .
- Must have a transition defined from each state for every symbol in Σ .

Old State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3

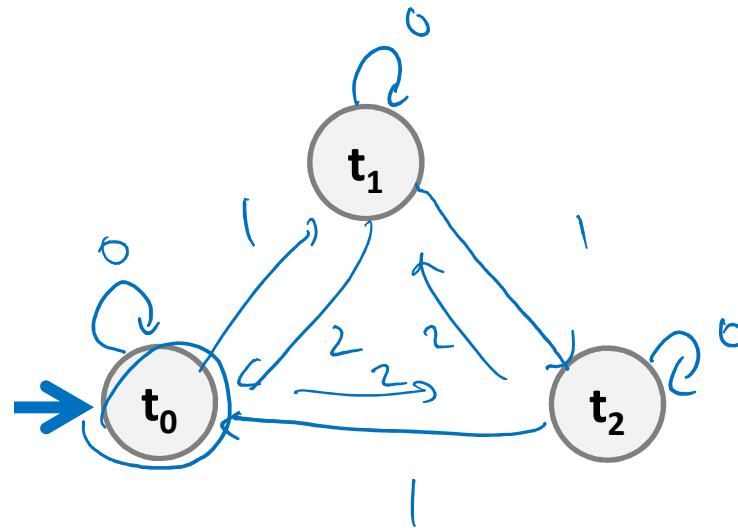


Strings over $\{0, 1, 2\}$

M_1 : Strings with an even number of 2's

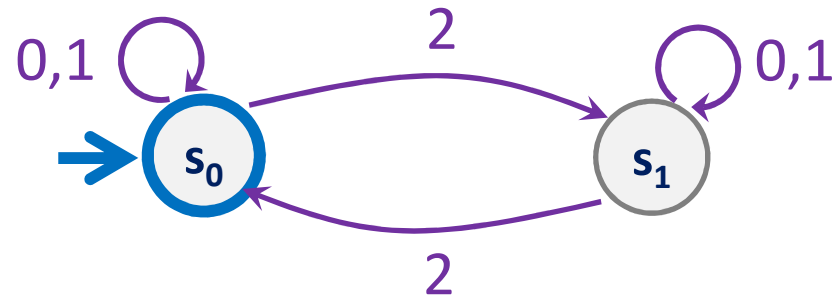


M_2 : Strings where the sum of digits mod 3 is 0

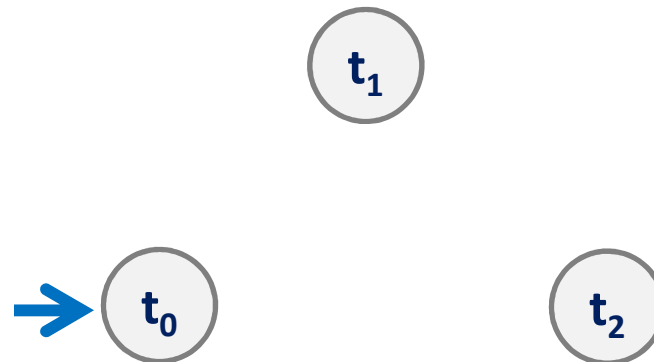


Strings over $\{0, 1, 2\}$

M_1 : Strings with an even number of 2's

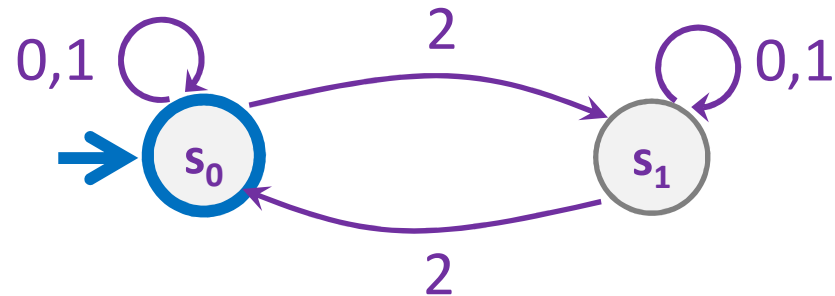


M_2 : Strings where the sum of digits mod 3 is 0

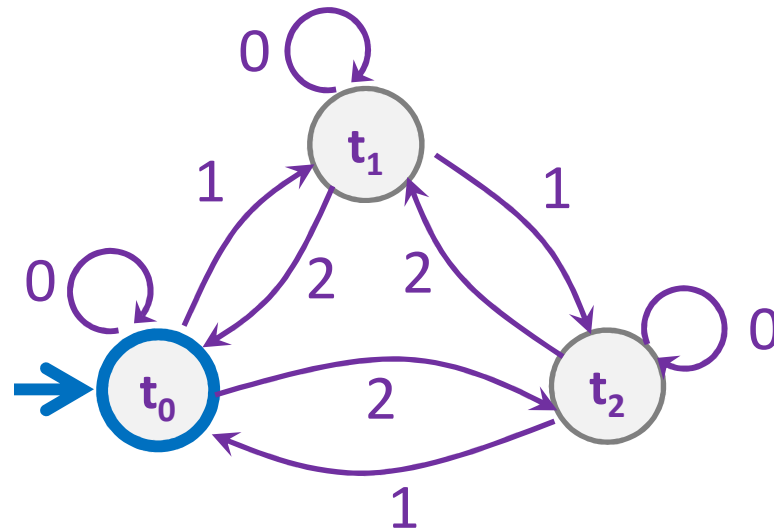


Strings over $\{0, 1, 2\}$

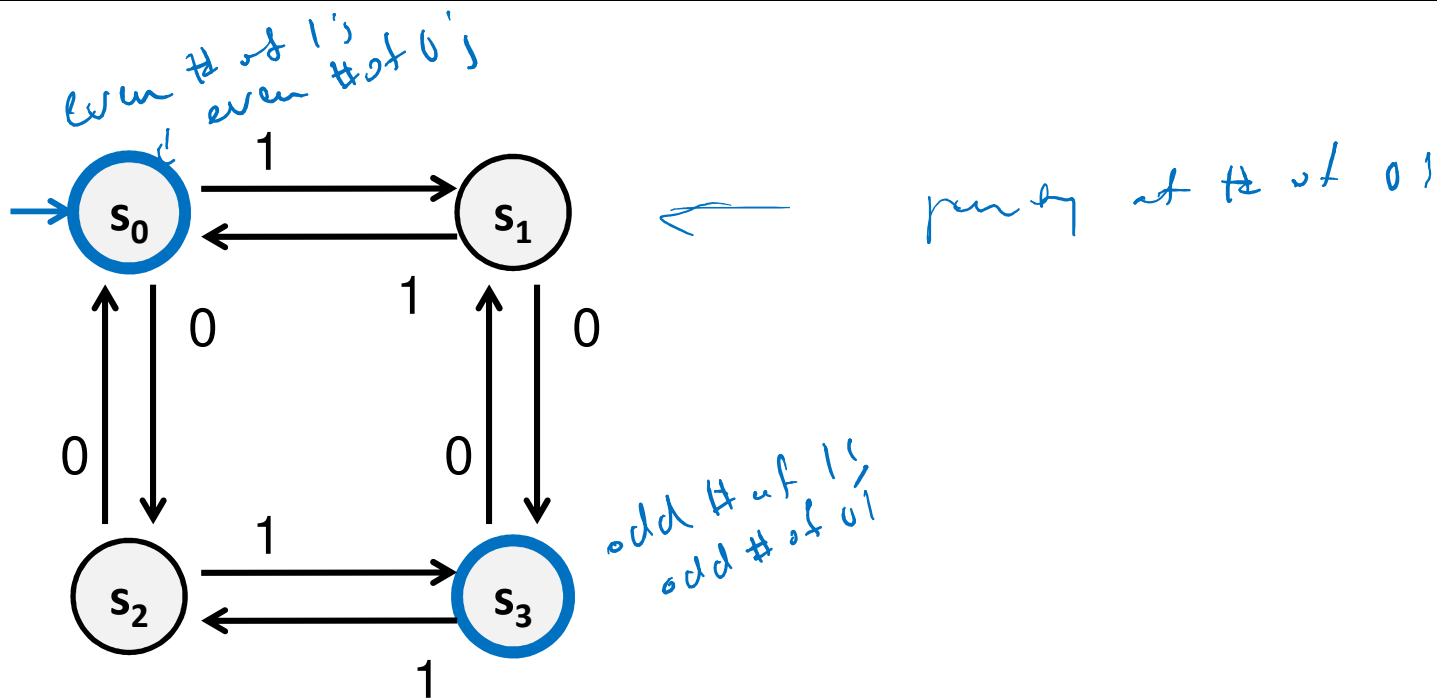
M_1 : Strings with an even number of 2's



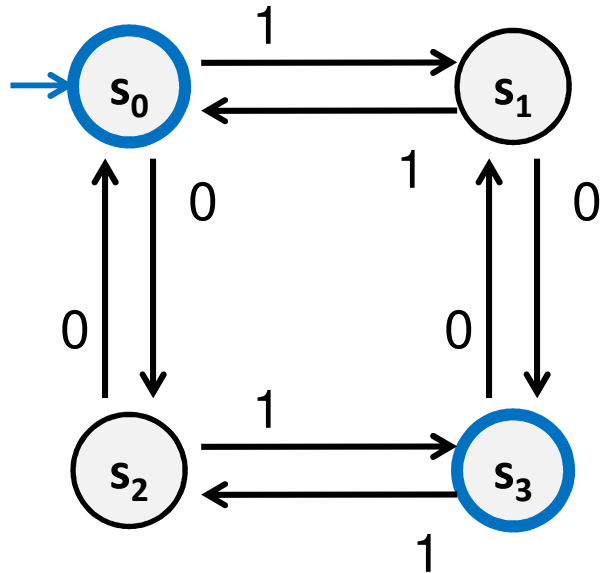
M_2 : Strings where the sum of digits mod 3 is 0



What language does this machine recognize?



What language does this machine recognize?

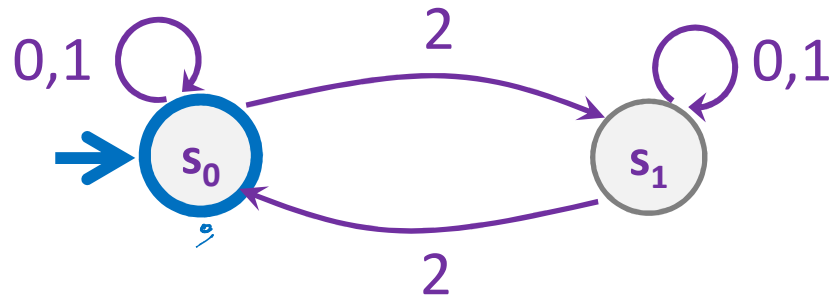


The set of all binary strings with # of 1's \equiv # of 0's (mod 2)
(both are even or both are odd).

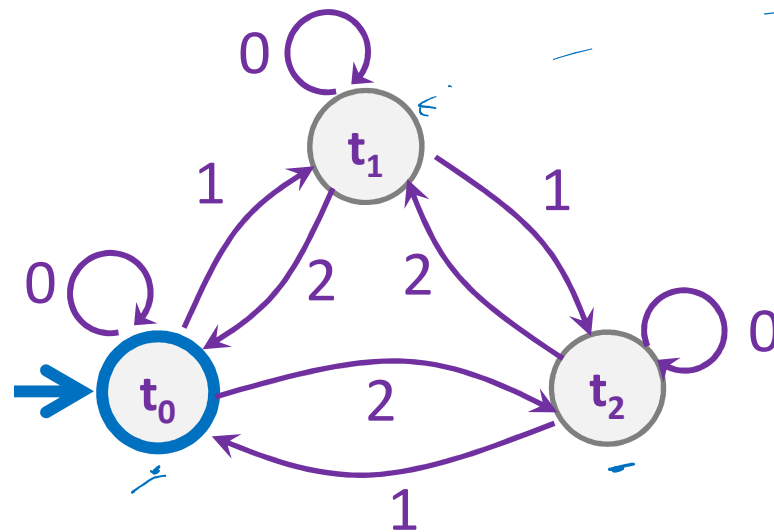
All strings of even length

Strings over $\{0, 1, 2\}$

M_1 : Strings with an even number of 2's

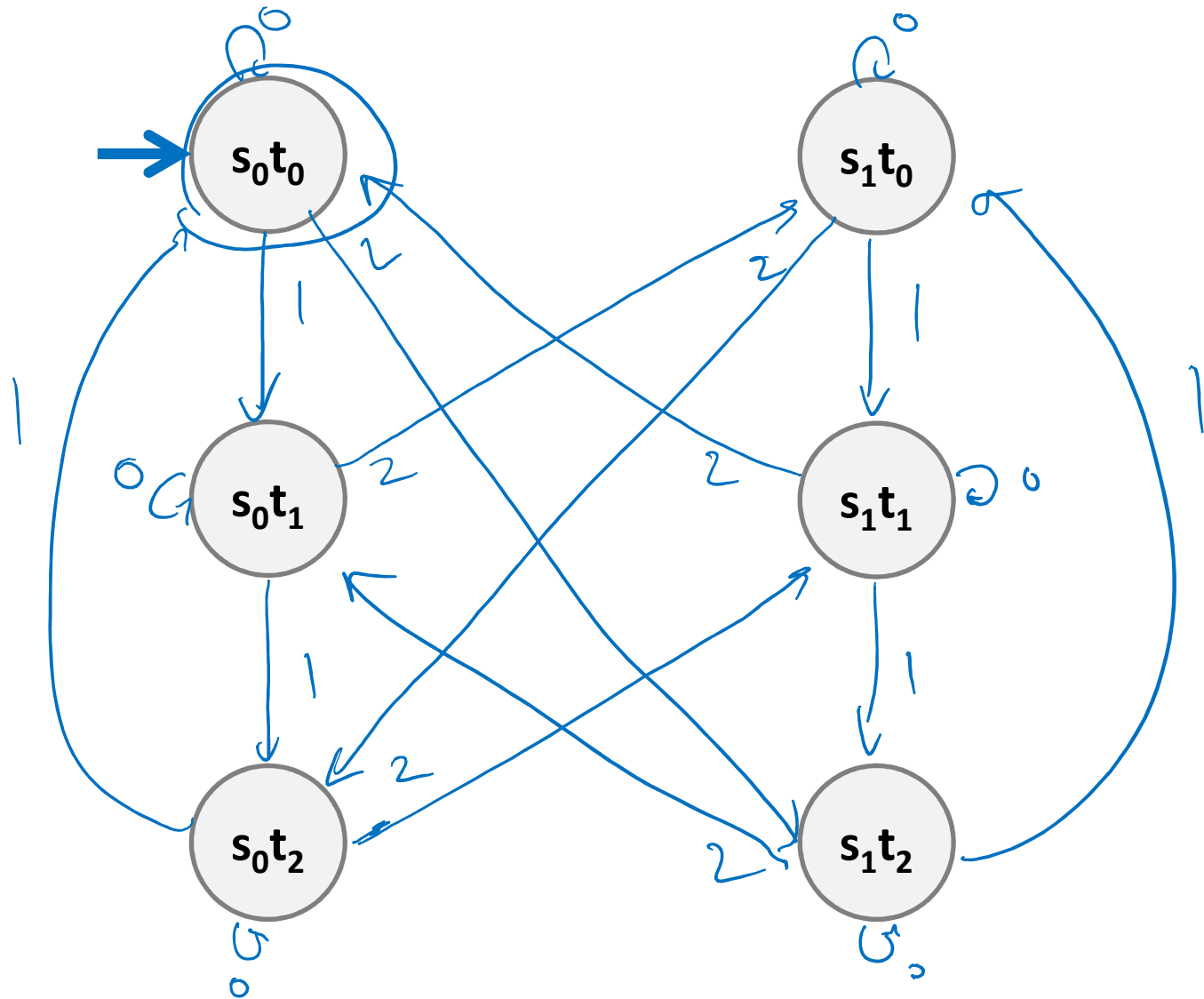


M_2 : Strings where the sum of digits mod 3 is 0

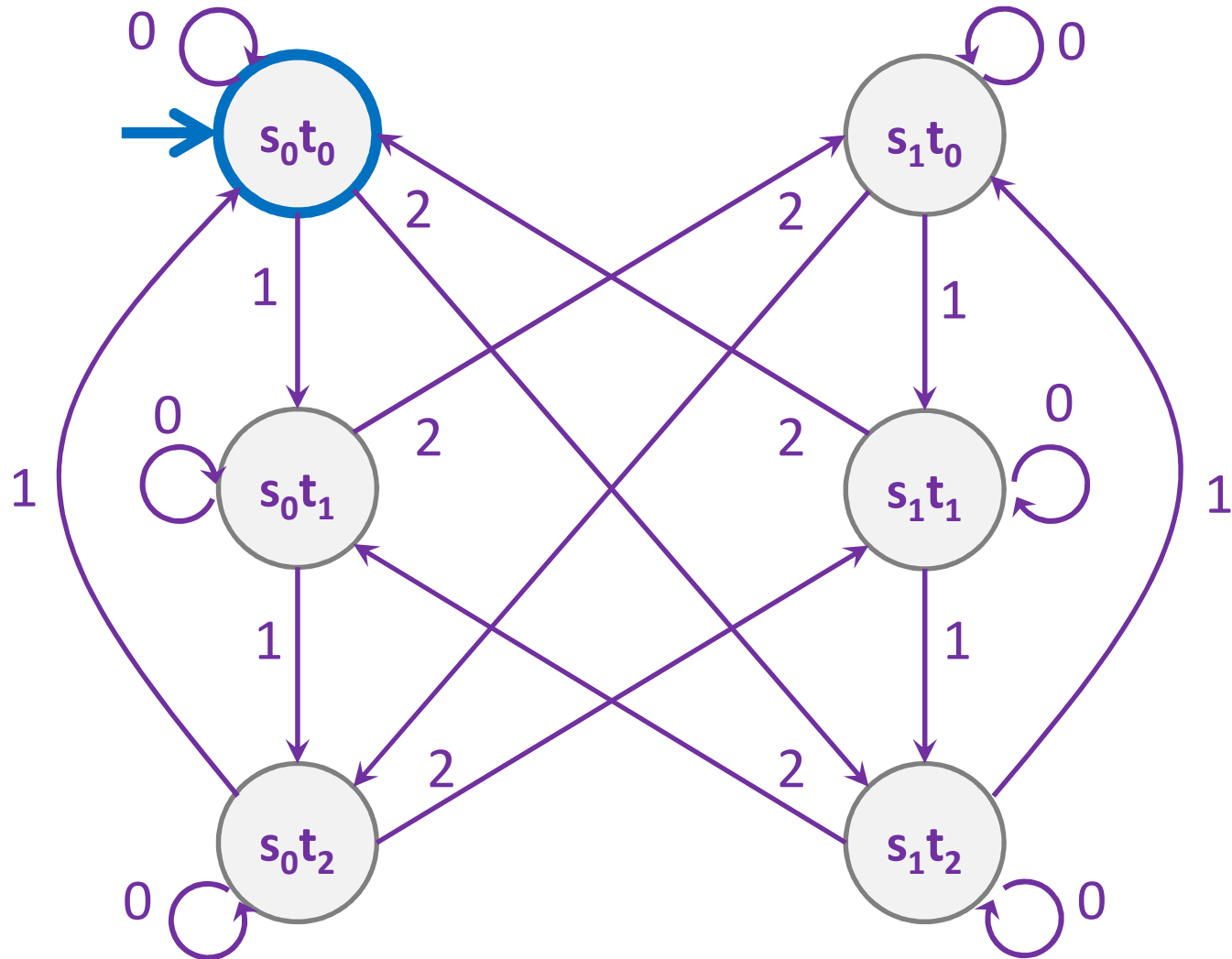


01212⁰

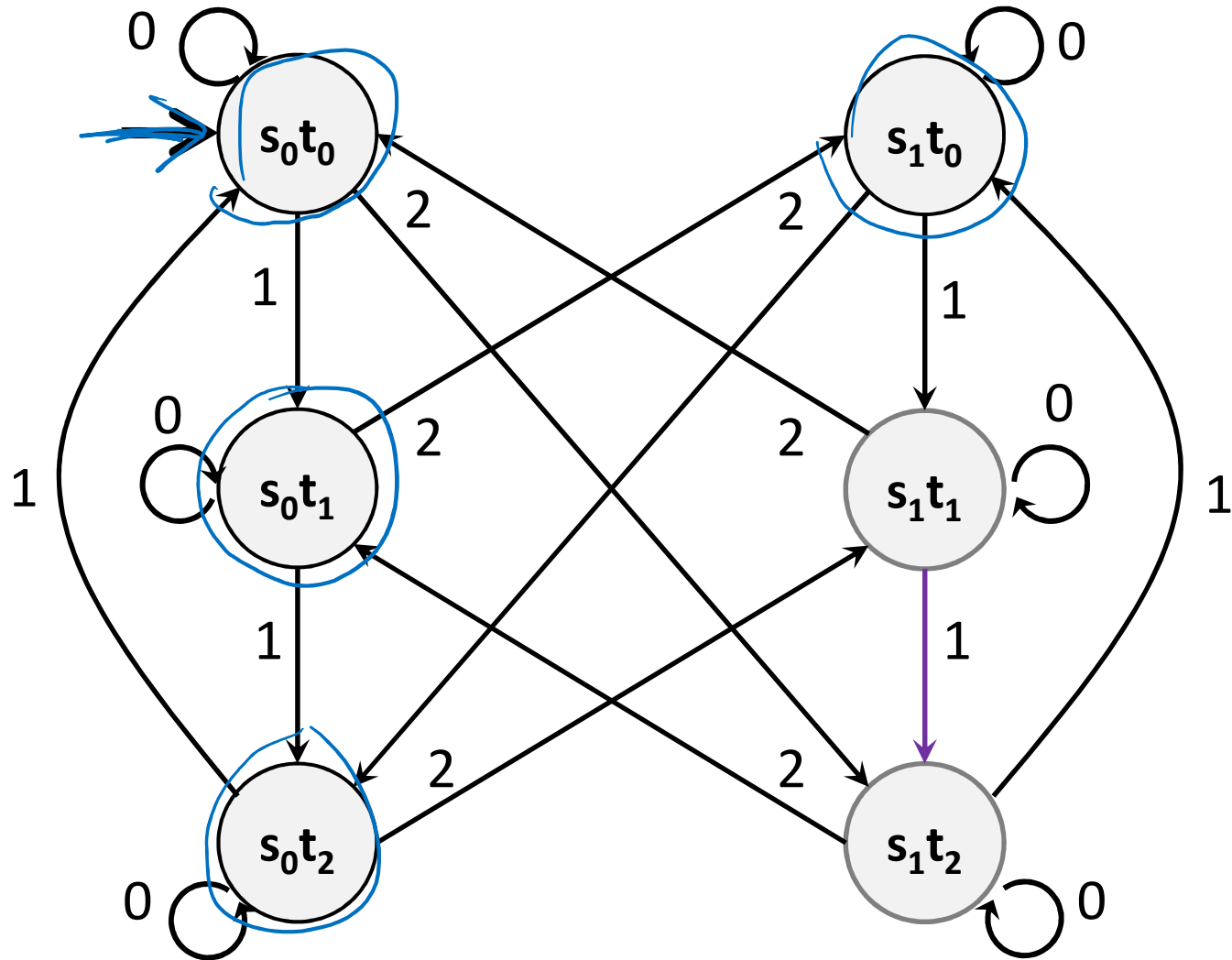
Strings over $\{0,1,2\}$ w/ even number of 2's and mod 3 sum 0



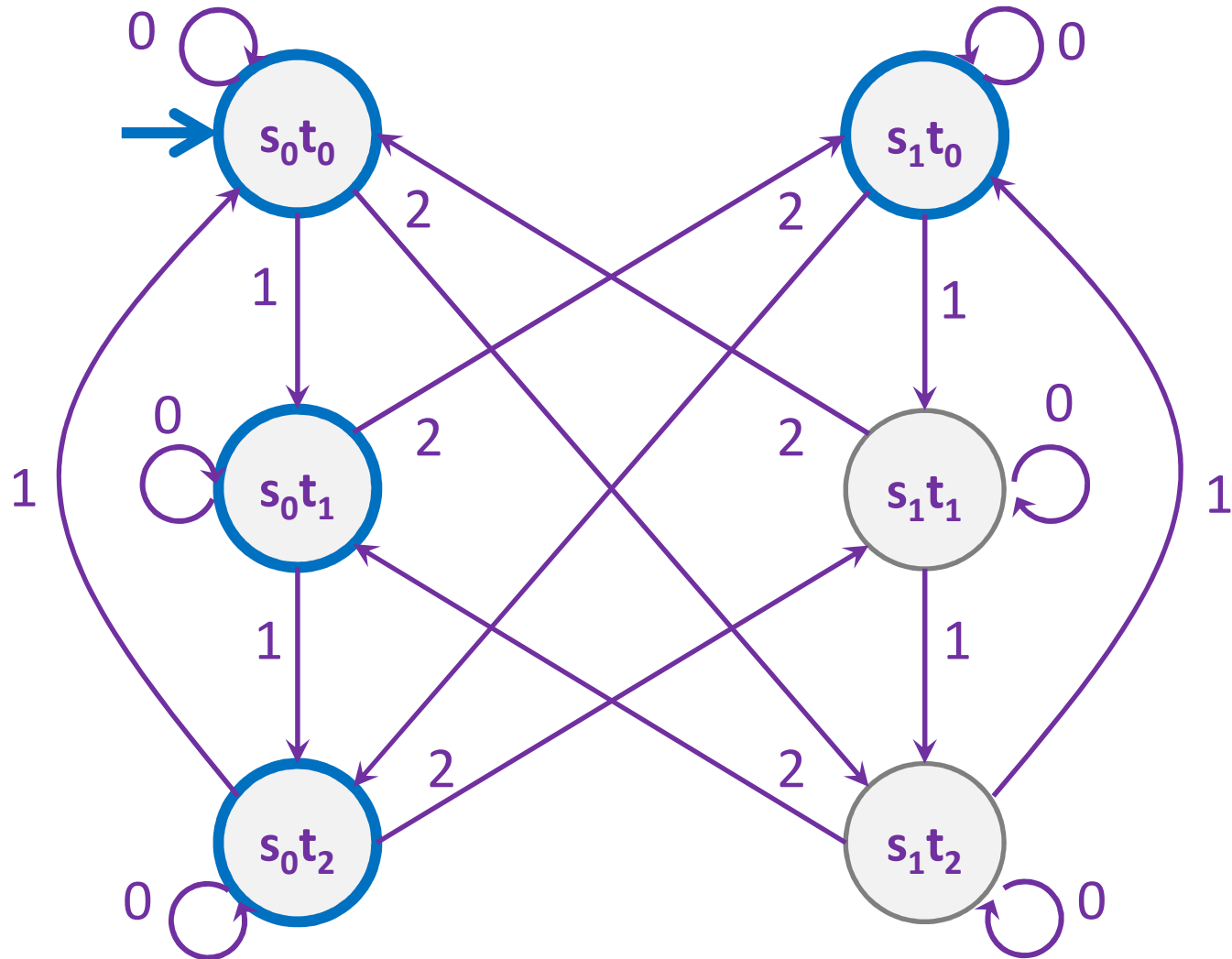
Strings over $\{0,1,2\}$ w/ even number of 2's and mod 3 sum 0



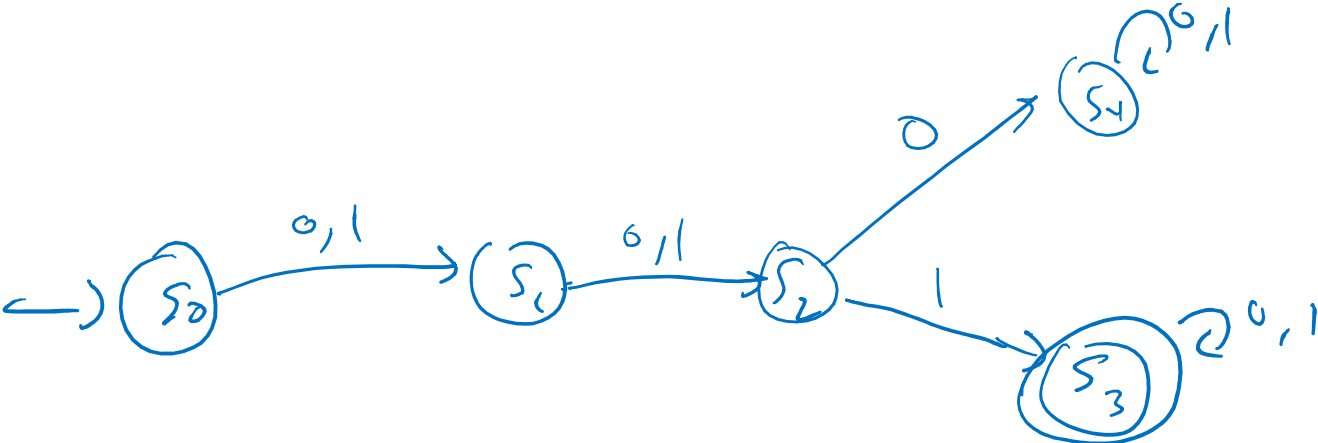
Strings over $\{0,1,2\}$ w/ even number of 2's OR mod 3 sum 0?



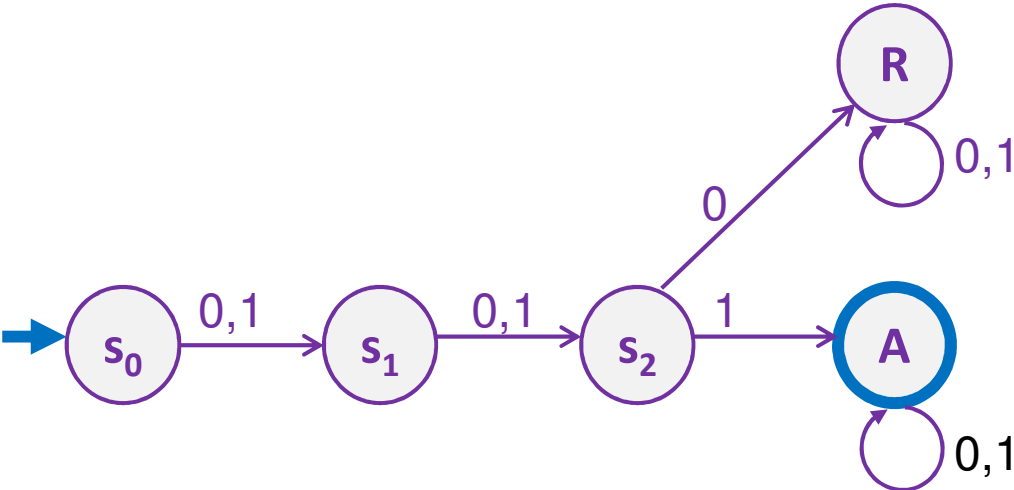
Strings over $\{0,1,2\}$ w/ even number of 2's OR mod 3 sum 0



The set of binary strings with a 1 in the 3rd position from the start

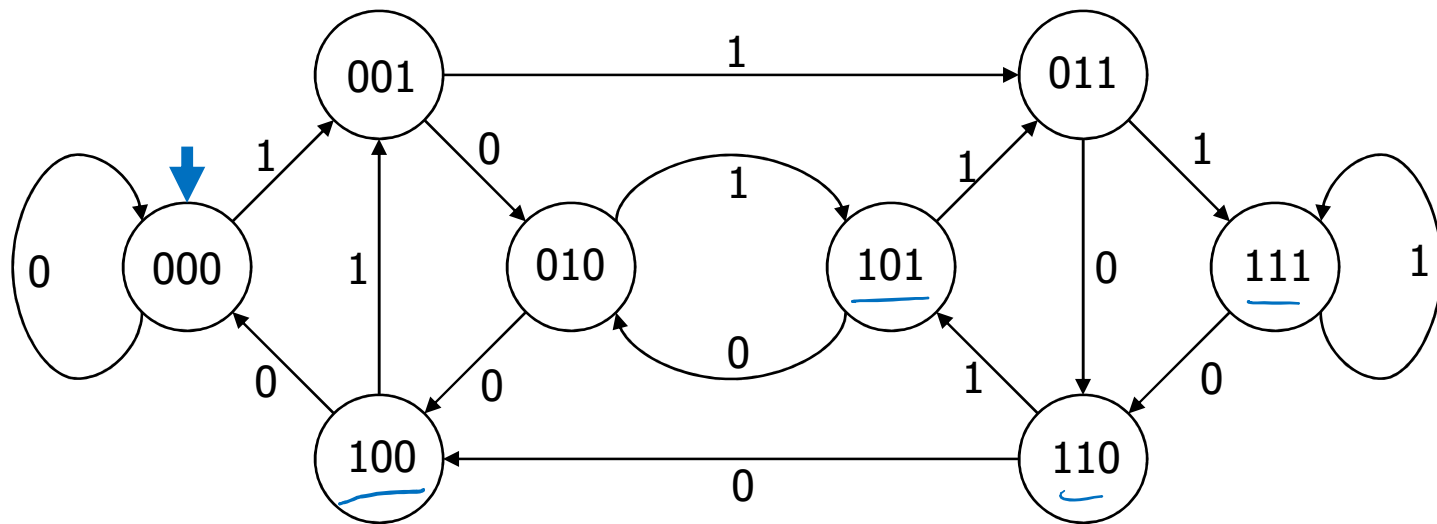


The set of binary strings with a 1 in the 3rd position from the start

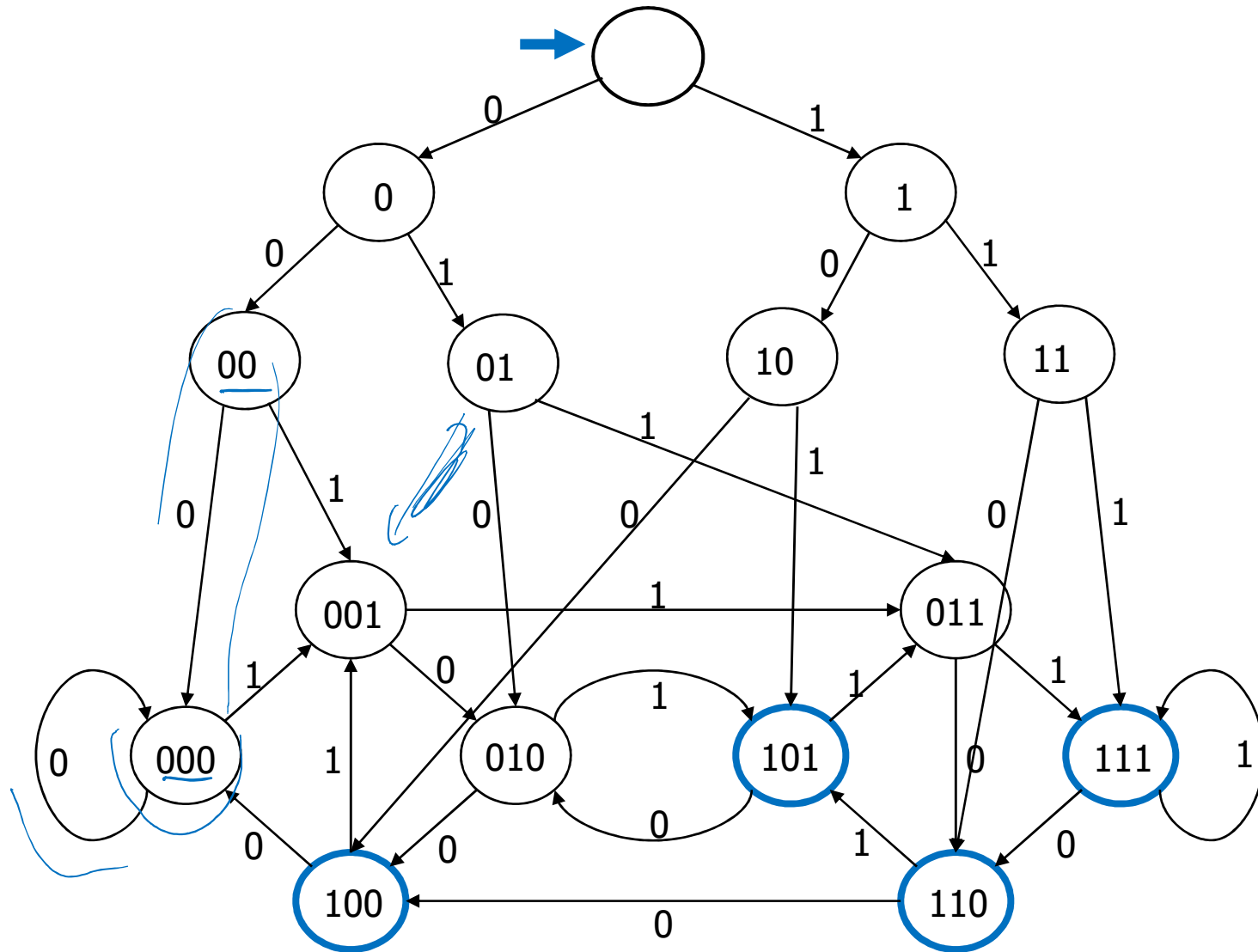


The set of binary strings with a **1** in the **3rd** position from the **end**

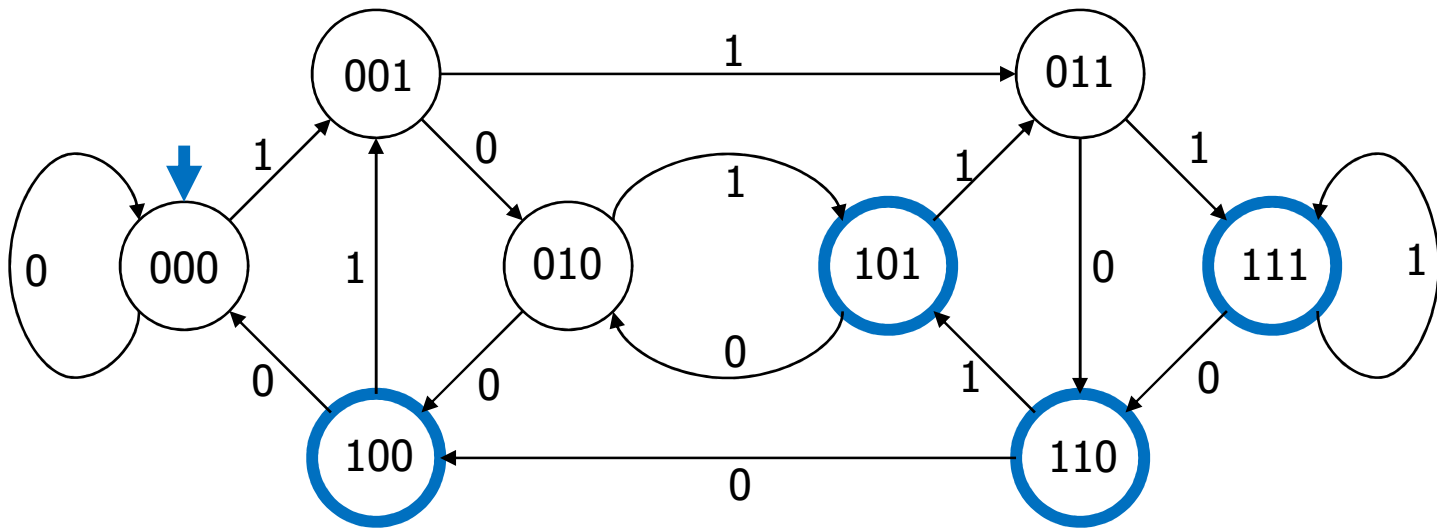
3 bit shift register “Remember the last three bits”



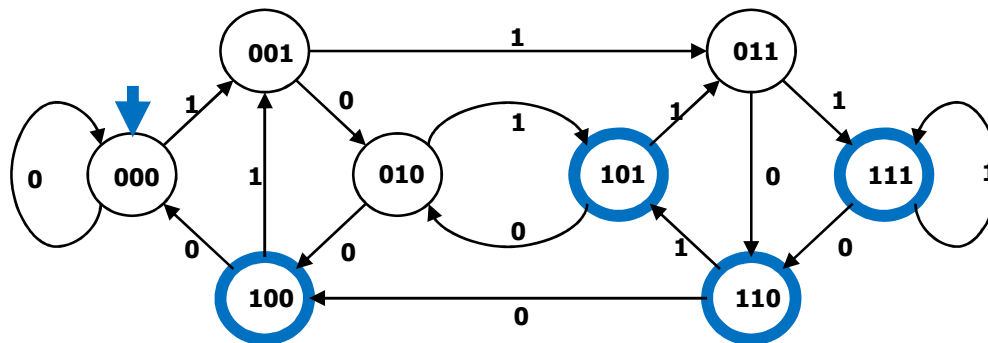
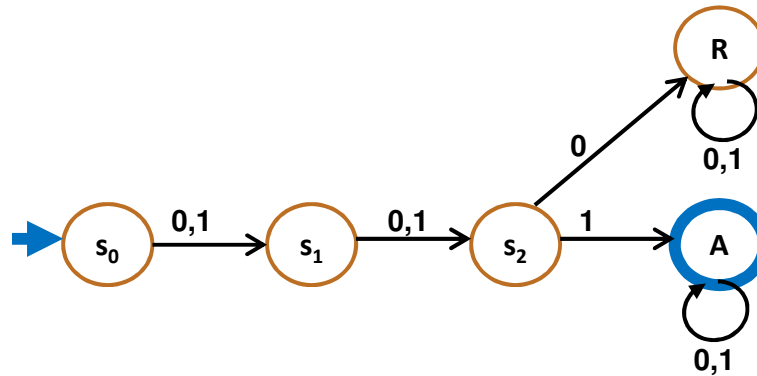
The set of binary strings with a 1 in the 3rd position from the end



The set of binary strings with a 1 in the 3rd position from the end



The beginning versus the end



Adding Output to Finite State Machines

- So far we have considered finite state machines that just accept/reject strings
 - called “Deterministic Finite Automata” or DFAs
- Now we consider finite state machines that with output
 - These are the kinds used as controllers



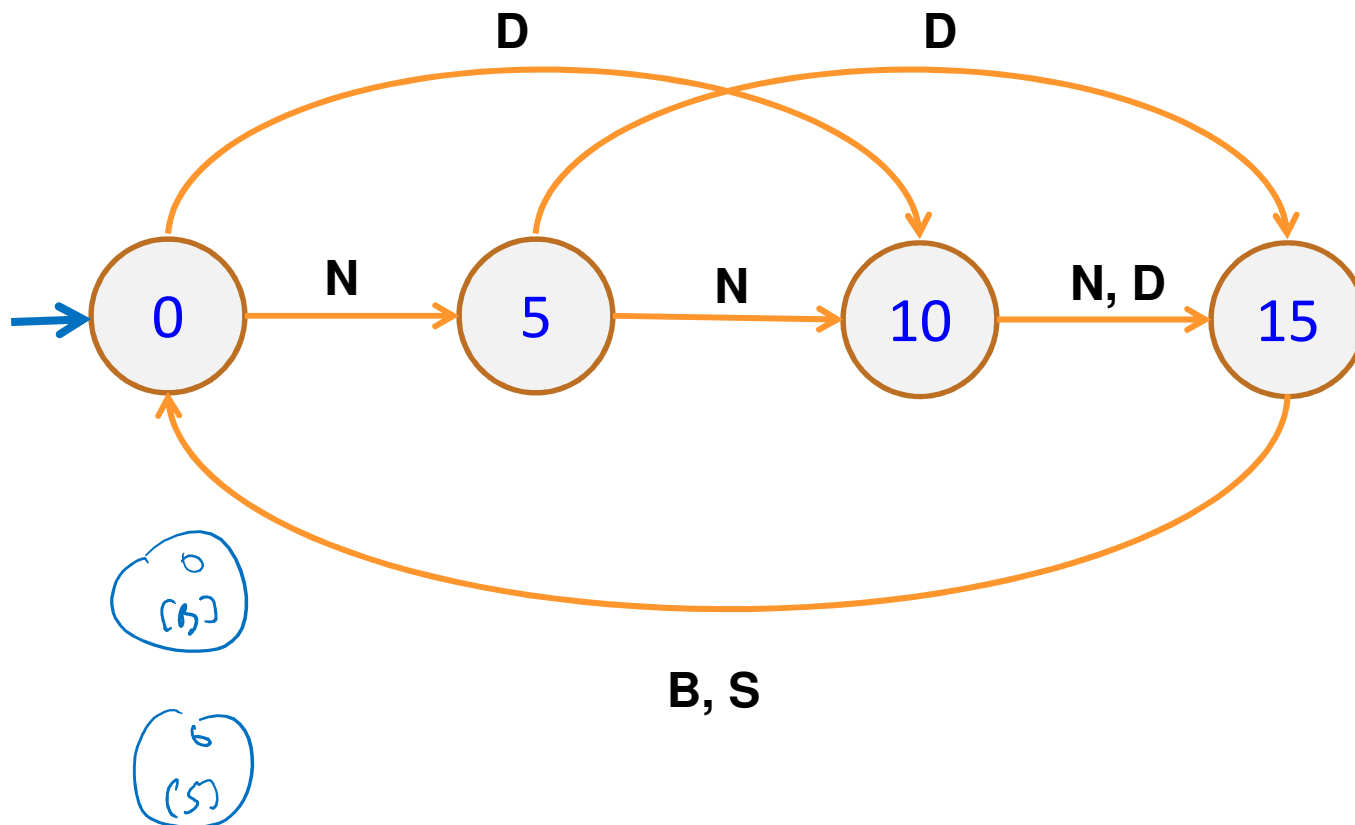
Vending Machine



Enter 15 cents in dimes or nickels
Press S or B for a candy bar

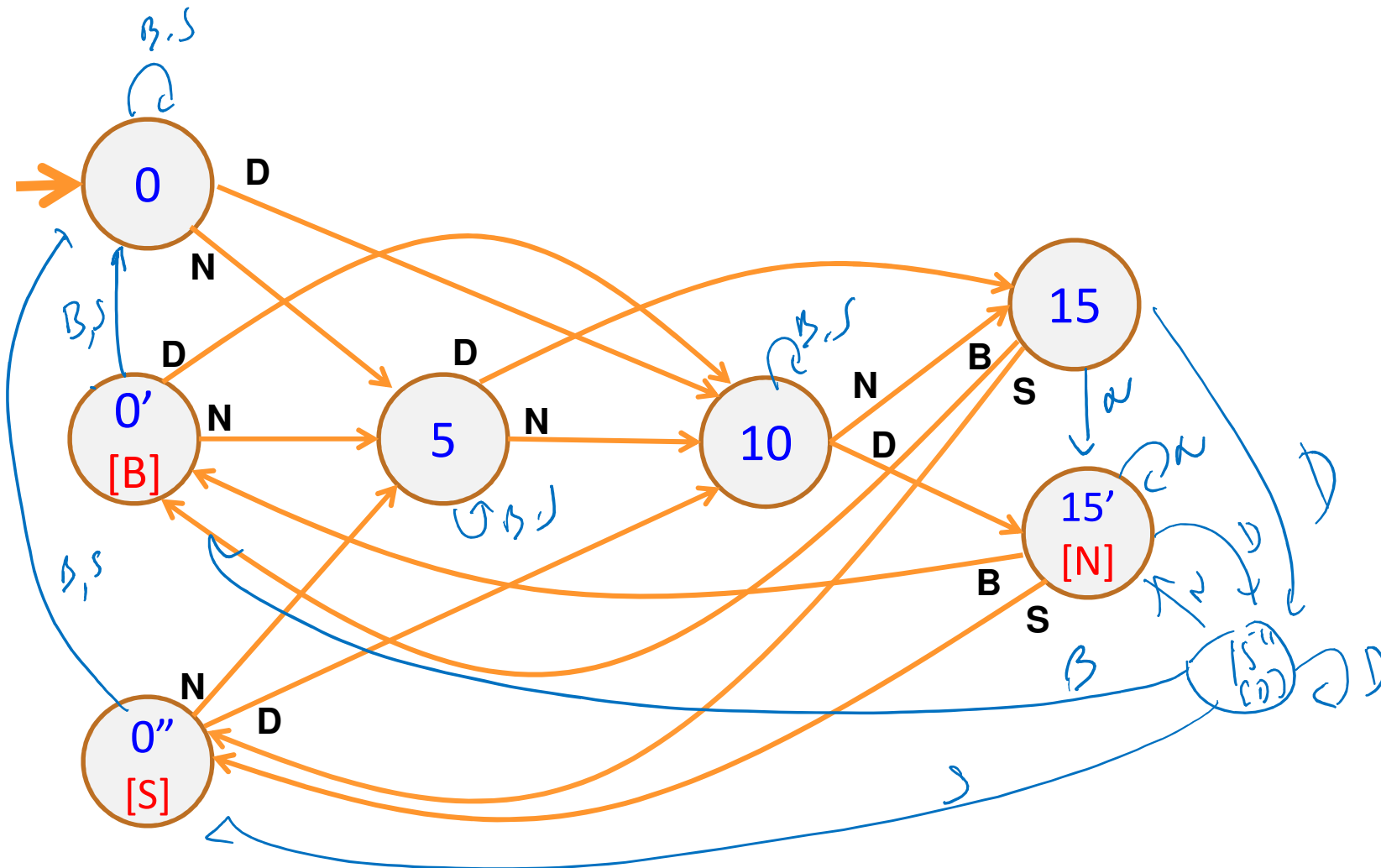


Vending Machine, v0.1



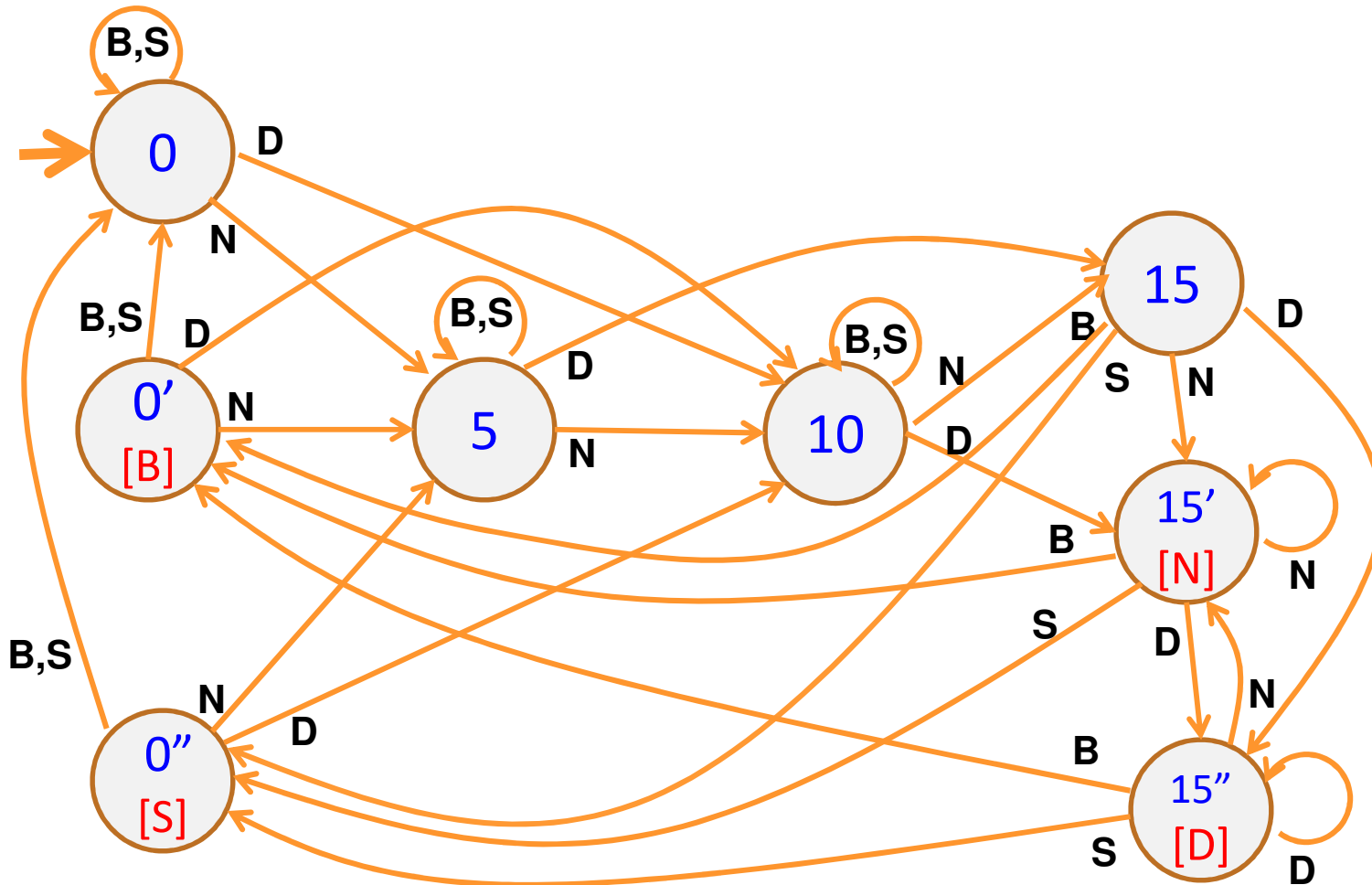
Basic transitions on **N** (nickel), **D** (dime), **B** (butterfinger), **S** (snickers)

Vending Machine, v0.2



Adding output to states: **N** – Nickel, **S** – Snickers, **B** – Butterfinger

Vending Machine, v1.0



Adding additional “unexpected” transitions to cover all symbols for each state