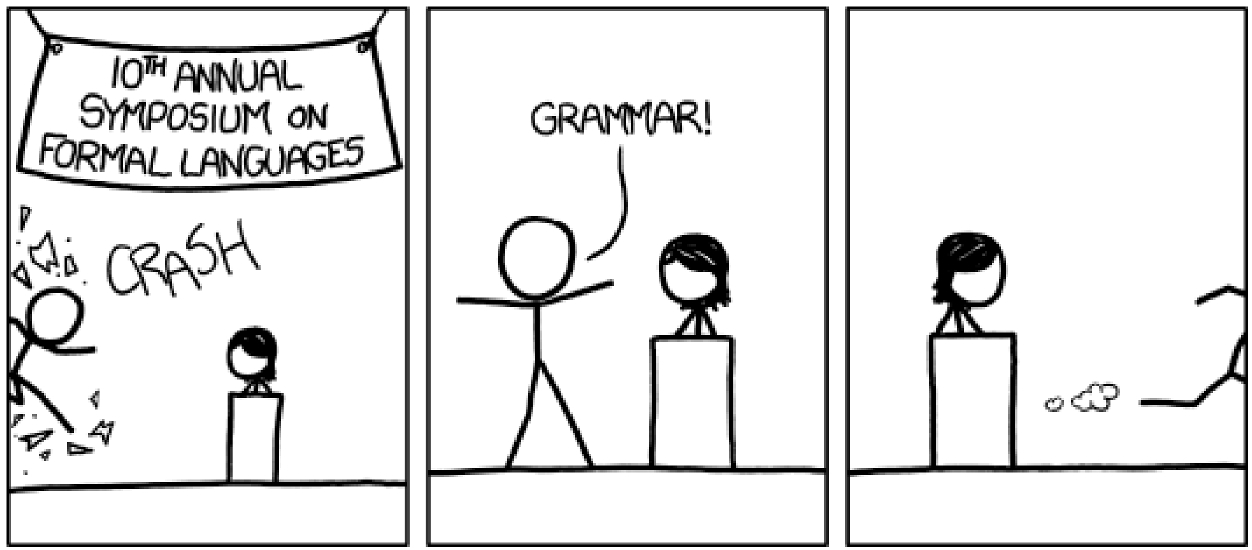
CSE 311: Foundations of Computing

Lecture 19: Regular Expressions & Context-Free Grammars

Recieved < 5 for 2a of midterm, write Regrade Prob 2



[Audience looks around]
“What is going on? There must be some context we’re missing”

Review: each regular expression is a "pattern"

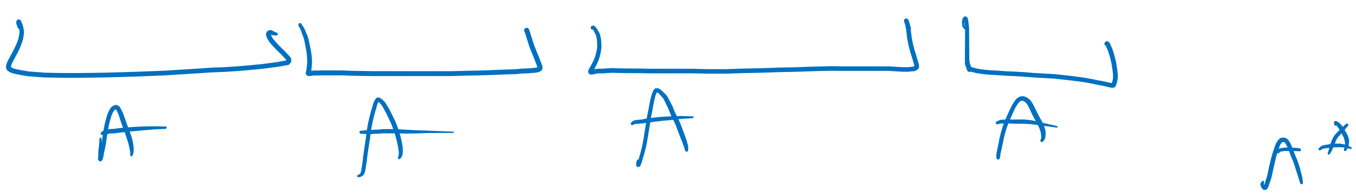
ε matches the **empty string**

a matches the one character string *a*

(A ∪ B) matches all strings that either **A** matches or **B** matches (or both)

(AB) matches all strings that have a first part that **A** matches followed by a second part that **B** matches

A* matches all strings that have any number of strings (even 0) that **A** matches, one after another



Examples

$\{0, 1\}$

- All binary strings that have an even # of 1's

$0^*(0^*10^*10^*)^*0^*$ ✓

$(0 \cup (10^*1))^*$ ✓

$11011 \in 0^*(10^*1)^*0^*$

$11011 \in (0^*10^*10^*)^*$

- All binary strings that *don't* contain 101

$111 \notin (0^*110^*)^*$

$111 \notin 0^*(1^*000^*1^*)^*$

$11011 \overset{?}{\in} 0^*((1^*000^*1^*)^* \cup 1^*)^*0^*$

Examples

- All binary strings that have an even # of 1's

  e.g., $0^*(10^*10^*)^*$

- All binary strings that *don't* contain 101

  e.g., $0^*(1 \cup 000^*)^* 0^*$

Limitations of Regular Expressions

- Not all languages can be specified by regular expressions
- Even some easy things like
  - Palindromes
  - Strings with equal number of 0's and 1's
- But also more complicated structures in programming languages
  - Matched parentheses
  - Properly formed arithmetic expressions
  - etc.

Context-Free Grammars

- A Context-Free Grammar (CFG) is given by a finite set of substitution rules involving
  - A finite set **V** of *variables* that can be replaced
  - Alphabet **Σ** of *terminal symbols* that can't be replaced
  - One variable, usually **S**, is called the *start symbol*

- The rules involving a variable **A** are written as

$$\mathbf{A} \rightarrow w_1 \mid w_2 \mid \cdots \mid w_k$$

where each w_i is a string of variables and terminals – that is $w_i \in (\mathbf{V} \cup \boldsymbol{\Sigma})^*$

How CFGs generate strings

- Begin with start symbol **S**   S
- If there is some variable **A** in the current string you can replace it by one of the w's in the rules for **A**
  - $\mathbf{A} \rightarrow w_1 \mid w_2 \mid \cdots \mid w_k$
  - Write this as $x\mathbf{A}y \Rightarrow xwy$
  - Repeat until no variables left
- The set of strings the CFG generates are all strings produced in this way that have no variables

1^* $S \rightarrow \varepsilon \mid S1$

$S \rightarrow A$ $A \rightarrow \varepsilon \mid A1$

Example Context-Free Grammars

Example: $\mathbf{S} \rightarrow 0\mathbf{S}0 \mid 1\mathbf{S}1 \mid 0 \mid 1 \mid \varepsilon$

0101

$S \Rightarrow 0S0 \rightarrow 01S10 \rightarrow 0110$

Palindromes.

Example: $\mathbf{S} \rightarrow 0\mathbf{S} \mid \mathbf{S}1 \mid \varepsilon$

$S \Rightarrow S1 \Rightarrow 1$

$0^* 1^*$

Example Context-Free Grammars

Example: $\mathbf{S} \rightarrow 0\mathbf{S}0 \mid 1\mathbf{S}1 \mid 0 \mid 1 \mid \varepsilon$

The set of all binary palindromes

Example: $\mathbf{S} \rightarrow 0\mathbf{S} \mid \mathbf{S}1 \mid \varepsilon$

0*1*

Example Context-Free Grammars

Grammar for $\{0^n 1^n : n \geq 0\}$

(all strings with same # of 0's and 1's with all 0's before 1's)

$S \rightarrow 0S1 \mid \varepsilon$.

Example: $S \rightarrow (S) \mid SS \mid \varepsilon$

$\{(,)\}$

All correctly nested par–

((())) ()

$S \Rightarrow SS \rightarrow (S)S \Rightarrow ((S))S \Rightarrow (((S)))S \Rightarrow ((()))S$

$\Rightarrow ((()))(S) \Rightarrow ((()))()$

Example Context-Free Grammars

Grammar for $\{0^n 1^n : n \geq 0\}$

(all strings with same # of 0's and 1's with all 0's before 1's)

S $\rightarrow$ 0**S**1 | ε

Example: **S** $\rightarrow$ **(S) | SS** | ε

The set of all strings of matched parentheses

Simple Arithmetic Expressions

E→ E+E | E∗E | (E) | x | y | z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Generate (2∗x) + y

$E \Rightarrow E+E \Rightarrow (E)+E \Rightarrow (E \times E)+E \Rightarrow (2 \times E)+E \Rightarrow (2 \times x)+E \Rightarrow (2 \times x)+y$

Generate x+y∗z in two fundamentally different ways

$E \Rightarrow E+E \Rightarrow E + E \times E \Rightarrow x + E \times E \Rightarrow x + y \times E \Rightarrow x+y \times z$

$E \Rightarrow E \times E \Rightarrow E \times z \Rightarrow E + E \times z \Rightarrow x + E \times z \Rightarrow x+y \times z.$

Simple Arithmetic Expressions

$$E \rightarrow E+E \mid E*E \mid (E) \mid x \mid y \mid z \mid 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

Generate $(2*x) + y$

$$E \Rightarrow E+E \Rightarrow (E)+E \Rightarrow (E*E)+E \Rightarrow (2*E)+E \Rightarrow (2*x)+E \Rightarrow (2*x)+y$$

Generate $x+y*z$ in two fundamentally different ways

$$E \Rightarrow E+E \Rightarrow x+E \Rightarrow x+E*E \Rightarrow x+y*E \Rightarrow x+y*z$$

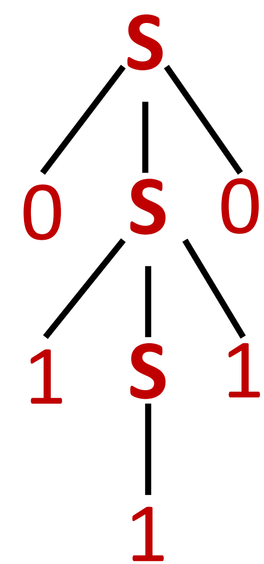
$$E \Rightarrow E*E \Rightarrow E+E*E \Rightarrow x+E*E \Rightarrow x+y*E \Rightarrow x+y*z$$

Parse Trees

Suppose that grammar **G** generates a string **x**

- A *parse tree* of **x** for **G** has
  - Root labeled **S** (start symbol of **G**)
  - The children of any node labeled **A** are labeled by symbols of **w** left-to-right for some rule $A \rightarrow w$
  - The symbols of **x** label the leaves ordered left-to-right

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$$



Parse tree of 01110

CFGs and recursively-defined sets of strings

- A CFG with the start symbol **S** as its only variable recursively defines the set of strings of terminals that **S** can generate

- A CFG with more than one variable is a simultaneous recursive definition of the sets of strings generated by *each* of its variables
  - Sometimes necessary to use more than one

building precedence in simple arithmetic expressions

- **E** – expression (start symbol)
- **T** – term **F** – factor **I** – identifier **N** - number

$$\mathbf{E} \rightarrow \mathbf{T} \mid \mathbf{E}+\mathbf{T}$$

$$\mathbf{T} \rightarrow \mathbf{F} \mid \mathbf{F}*\mathbf{T}$$

$$\mathbf{F} \rightarrow (\mathbf{E}) \mid \mathbf{I} \mid \mathbf{N}$$

$$\mathbf{I} \rightarrow x \mid y \mid z$$

$$\mathbf{N} \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

Backus-Naur Form (The same thing…)

BNF (Backus-Naur Form) grammars

- Originally used to define programming languages
- Variables denoted by long names in angle brackets, e.g.

  <identifier>, <if-then-else-statement>, <assignment-statement>, <condition>

  $::=$ used instead of $\rightarrow$

BNF for C

```
statement:
   ((identifier | "case" constant-expression | "default") ":")*
   (expression? ";" |
    block |
    "if" "(" expression ")" statement |
    "if" "(" expression ")" statement "else" statement |
    "switch" "(" expression ")" statement |
    "while" "(" expression ")" statement |
    "do" statement "while" "(" expression ")" ";" |
    "for" "(" expression? ";" expression? ";" expression? ")" statement |
    "goto" identifier ";" |
    "continue" ";" |
    "break" ";" |
    "return" expression? ";"
   )

block: "{" declaration* statement* "}"

expression:
  assignment-expression%

assignment-expression: (
    unary-expression (
      "=" | "*=" | "/=" | "%=" | "+=" | "-=" | "<<=" | ">>=" | "&=" |
      "^=" | "|="
    )
  )* conditional-expression

conditional-expression:
  logical-OR-expression ( "?" expression ":" conditional-expression )?
```

Parse Trees

Back to middle school:

<sentence>::=<noun phrase><verb phrase>

<noun phrase>::==<article><adjective><noun>

<verb phrase>::=<verb><adverb>|<verb><object>

<object>::=<noun phrase>

Parse:

The yellow duck squeaked loudly

The red truck hit a parked car