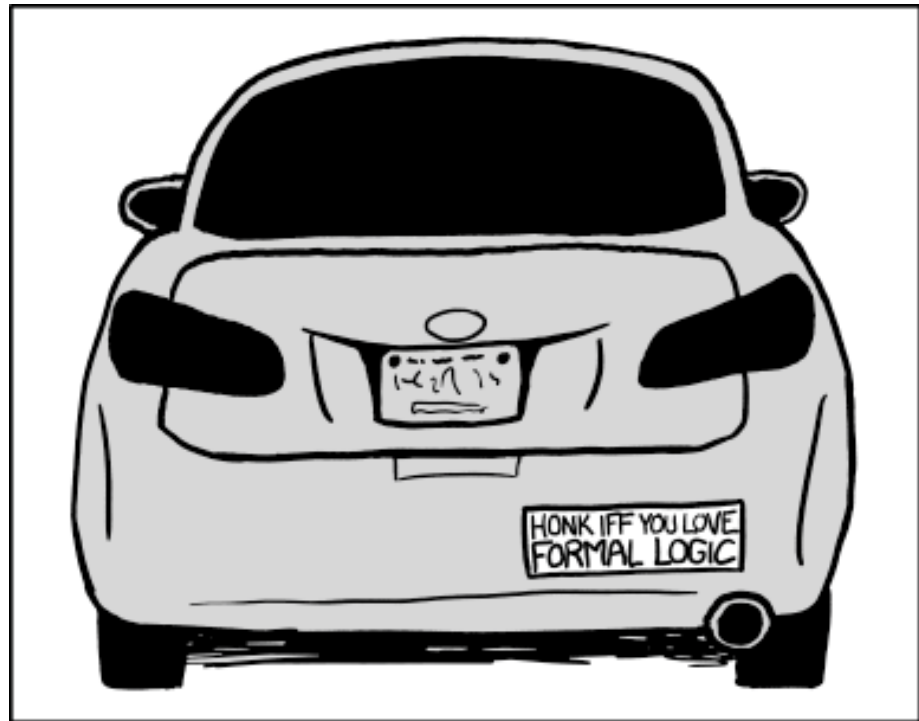


# CSE 311: Foundations of Computing I

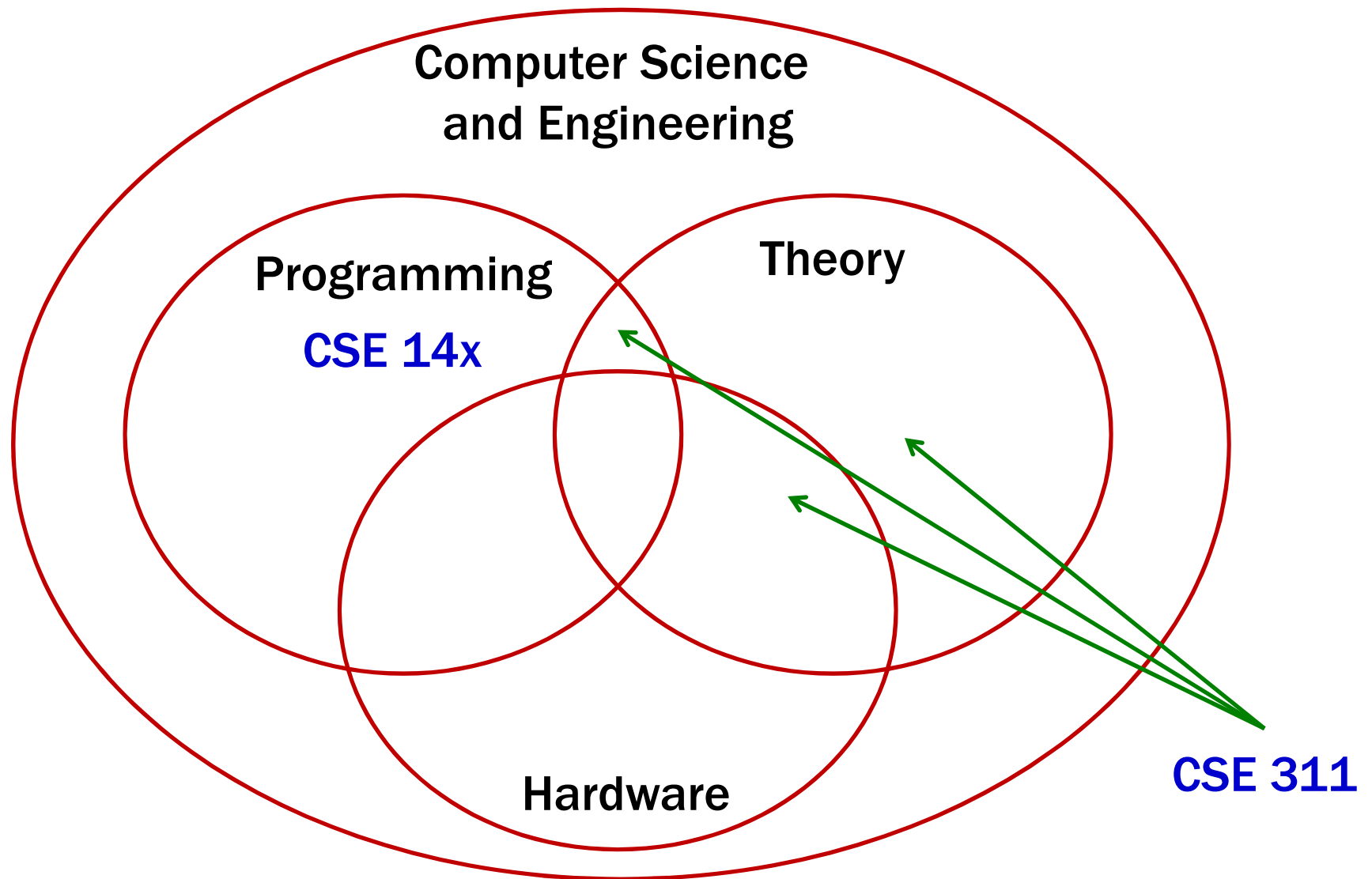
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## Lecture 1: Propositional Logic



# Some Perspective

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# About the Course

---

**We will study the *theory* needed for CSE:**

## **Logic:**

How can we describe ideas *precisely*?

## **Formal Proofs:**

How can we be *positive* we're correct?

## **Number Theory:**

How do we keep data *secure*?

## **Relations/Relational Algebra:**

How do we store information?

## **Finite State Machines:**

How do we design hardware and software?

## **Turing Machines:**

Are there problems computers *can't* solve?

# About the Course

---

**It's about perspective!**

- **Tools for reasoning about difficult problems**
- **Tools for communicating ideas, methods, objectives...**
- **Tools for automating difficult problems**
- **Fundamental structures for computer science**

# About the Course

---

**It's about perspective!**

- Tools for reasoning about difficult problems
- Tools for communicating ideas, methods, objectives...
- Tools for automating difficult problems
- Fundamental structures for computer science

**This is NOT a programming course!**

# Instructors

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## Paul Beame



**Section A**  
**MWF 10:30-11:20 in GUG 220**

**Office Hours:**  
**MWF 11:30-12:00 and TBA**  
**CSE 668**

## Shayan Oveis Gharan



**Section B**  
**MWF 1:30-2:20 in EEB 125**

**Office Hours:**  
**MWF 2:30-3:00 and TBA**  
**CSE 636**

**Office hours are for students in both sections**

# TAs and Administrivia

---

## Teaching Assistants:

Jiechen Chen

Joshua Fan

Wei Lin

Kaidi Pei

Jefferson Van Wagenen

Simone Zhang

Jie Du

Sarang Joshi

Evan McCarty

Michelle Prawiro

Laura Vonessen

Kaiyu Zheng

## Homework:

**Due WED at 6 pm online**

**Write up individually**

**Extra Credit**

## Grading (roughly):

**50% Homework**

**15-20% Midterm**

**30-35% Final Exam**

## Section:

**Thursdays**

**– starting tomorrow!**

**Office Hours: TBA**

**(Optional) Book:**

**Rosen: Readings for 6<sup>th</sup> (used) or  
7<sup>th</sup> (cut down) editions.**

**Good for practice with solved problems**

**Overload: <http://tinyurl.com/zlarys2>**

**All Course Information @ [cs.uw.edu/311](http://cs.uw.edu/311)**

# Administrivia

## CSE 311: Foundations of Computing I

Autumn, 2016

### Paul Beame

Section A: MWF 10:30-11:20, GUG 220  
Office hours: TBA, CSE 668

### Shayan Oveis Gharan

Section B: MWF 1:30-2:20, EEB 125  
Office hours: TBA, CSE 636

### Email and discussion:

Class email list: [cse311\\_aul5](#) [[archives](#)]

Please send any e-mail about the course to [cse311-staff@cs](mailto:cse311-staff@cs.washington.edu).

[Discussion Board](#) (moderated by TBA)

Use this board to discuss the content of the course. That includes everything **except** the solutions to current homework problems. Feel free to discuss homeworks and exams from past incarnations of the course, and any confusion over topics discussed in class. It is also acceptable to ask for *clarifications* about the statement of homework problems, but not about their solutions.

### Textbook:

There is no required text for the course. Especially over the first 6-7 weeks of the course, the following textbook can be a useful companion: Rosen, *Discrete Mathematics and Its Applications*, McGraw-Hill. We will support two versions equally: (1) a special reduced version of the 7th edition which at \$60 costs < 1/4 of the ridiculous price of the full text, and (2) the 6th edition, which is available used through the bookstore for even less money. It should also be available on short-term loan from the Engineering Library.

### Course Calendar

#### Lectures

#	date	topic	slides	inked (A)	inked (B)	reading (Rosen)
1	Wed, Sep 28	Logic	<a href="#">pdf</a>			1.1, 1.2 (7th) 1.1 (6th)
2	Fri, Sep 30	Logic				1.1-1.3 (7th) 1.1-1.2 (6th)
3	Mon, Oct 3	Logic/Boolean Algebra				12.1-12.3 (7th) 11.1-11.3 (6th)
4	Wed, Oct 5	Boolean Algebra/Circuits				12.1-12.3 (7th) 11.1-11.3 (6th)
5	Fri, Oct 7	Predicate Logic				1.4-1.5 (7th) 1.3-1.4 (6th)
6	Mon, Oct 10	Predicate Logic, Inference				1.6-1.7 (7th) 1.5-1.7 (6th)
7	Wed, Oct 12	Logical Inference and Proofs				1.6-1.7 (7th) 1.5-1.7 (6th)
8	Fri, Oct 14	More Proofs				1.6-1.7 (7th) 1.5-1.7 (6th)



TA	Office hours	Room
Section	Day/Time	Room

### Homeworks [[Grading guidelines](#)]:

- **Homework #1**, due Wednesday, 3-Oct.

### Exams:

- **Midterm exam:**  
In class, Monday, 7-Nov-2016
- **Final exam:**  
Monday, 12-Dec-2016  
The **final exam has been rescheduled** so that both lectures can take a common exam. The times will be 2:30-4:20 pm (the original exam time for the 1:30 section) and 4:30-6:20 pm with location TBA. Students may take the exam at either time. We will send out a Catalyst survey to get an idea of the number of students taking it at each time.

All Course Information @ [cs.uw.edu/311](http://cs.uw.edu/311)



# Administrivia

## CSE 311: Foundations of Computing I

Autumn, 2016

### Paul Beame

Section A: MWF 10:30-11:20, GUG 220  
Office hours: TBA, CSE 668

### Shayan Oveis Gharan

Section B: MWF 1:30-2:20, EEB 125  
Office hours: TBA, CSE 636

### Email and discussion:

Class email list: [cse311\\_aul5](mailto:cse311_aul5) [[archives](#)]

Please send any e-mail about the course to [cse311-staff@cs](mailto:cse311-staff@cs).

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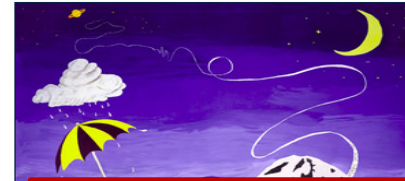
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8	Fri, Oct 14	More Proofs				1.6-1.7 (7th) 1.5-1.7 (6th)



**Midterm: Mon, Nov 7 in class**

**Final Exam: Mon, Dec 12**

- B section 2:30-4:20
- A section probably 4:30-6:20
  - **Not at 8:30-10:20 time in exam schedule**
- **Location TBA**

### Exams:

- **Midterm exam:**  
In class, Monday, 7-Nov-2016
- **Final exam:**  
Monday, 12-Dec-2016  
The **final exam has been rescheduled** so that both lectures can take a common exam. The times will be 2:30-4:20 pm (the original exam time for the 1:30 section) and 4:30-6:20 pm with location TBA. Students may take the exam at either time. We will send out a Catalyst survey to get an idea of the number of students taking it at each time.

All Course Information @ [cs.uw.edu/311](http://cs.uw.edu/311)

# Logic: The Language of Reasoning

---

**Why not use English?**

– Turn right here...

– Buffalo buffalo Buffalo buffalo buffalo  
buffalo Buffalo buffalo

– We saw her duck

# Logic: The Language of Reasoning

---

## Why not use English?

- Turn right here...

Does “right” mean the direction or now?

- Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo

This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.

- We saw her duck

Does “duck” mean the animal or crouch down?

# Logic: The Language of Reasoning

---

## Why not use English?

- Turn right here...

Does “right” mean the direction or now?

- Buffalo buffalo Buffalo buffalo buffalo  
buffalo Buffalo buffalo

This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.”

- We saw her duck

Does “duck” mean the animal or crouch down?

## “Language” like Java or English

- Words, sentences, paragraphs, arguments...
- Today is about *words* and *sentences*

# Why Learn A New Language?

---

**Logic, as the “language of reasoning”, will help us...**

- Be more precise**
- Be more concise**
- Figure out what a statement means more quickly**

# Propositions

---

A ***proposition*** is a statement that

- has a truth value, and
- is “well-formed”



“If I were to ask you out, would your answer to that question be the same as your answer to this one?”

# Are These Propositions?

---

$$2 + 2 = 5$$

The home page renders correctly in Chrome.

Turn in your homework on Wednesday.

This statement is false.

Akjsdf!

Who are you?

Every positive even integer can be written as the sum of two primes.

# Are These Propositions?

---

**$2 + 2 = 5$**

This is a proposition. It's okay for propositions to be false.

**The home page renders correctly in Chrome.**

This is a proposition. It's okay for propositions to be false.

**Turn in your homework on Wednesday.**

This is a "command" which means it doesn't have a truth value.

**This statement is false.**

This statement does not have a truth value! (If it's true, it's false, and vice versa.)

**Akjsdf!**

This is not a proposition because it's gibberish.

**Who are you?**

This is a question which means it doesn't have a truth value.

**Every positive even integer can be written as the sum of two primes.**

This is a proposition. We don't know if it's true or false, but we know it's one of them!



# Propositions

---

A ***proposition*** is a statement that

- has a truth value, and
- is “well-formed”

We need a way of talking about *arbitrary* ideas...

**Propositional Variables:**

**Truth Values:**

# Propositions

---

A **proposition** is a statement that

- has a truth value, and
- is “well-formed”

We need a way of talking about *arbitrary* ideas...

Propositional Variables:  $p, q, r, s, \dots$

Truth Values:

- **T** for true
- **F** for false

# A Proposition

---

“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either.”

We’d like to *understand* what this proposition means.

This is where logic comes in. There are pieces that appear multiple times in the phrase (e.g., “you can get measles”).

These are called **atomic propositions**. Let’s list them:

# A Proposition

---

“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either.”

We’d like to *understand* what this proposition means.

This is where logic comes in. There are pieces that appear multiple times in the phrase (e.g., “you can get measles”).

These are called **atomic propositions**. Let’s list them:

Measles: “You can get measles”

Mumps: “You can get mumps”

MMR: “You had the MMR vaccine”

# Putting Them Together

---

**“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either.”**

Measles: “You can get measles”

Mumps: “You can get mumps”

MMR: “You had the MMR vaccine”

**Now, we put these together to make the sentence:**

((Measles and Mumps) if not MMR) but (if MMR then not (Measles or Mumps))

((Measles and Mumps) if not MMR) and (if MMR then not (Measles or Mumps))

**This is the general idea, but now, let’s define our *formal language*.**

# Logical Connectives

---

Negation (not)  $\neg p$

Conjunction (and)  $p \wedge q$

Disjunction (or)  $p \vee q$

Exclusive Or  $p \oplus q$

Implication  $p \rightarrow q$

Biconditional  $p \leftrightarrow q$

# Logical Connectives

---

Negation (not)	$\neg p$
Conjunction (and)	$p \wedge q$
Disjunction (or)	$p \vee q$
Exclusive Or	$p \oplus q$
Implication	$p \rightarrow q$
Biconditional	$p \leftrightarrow q$

Measles:

“You can get measles”

Mumps:

“You can get mumps”

MMR:

“You had the MMR vaccine”

“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either.”



((Measles and Mumps) if not MMR) and (if MMR then not (Measles or Mumps))

# Logical Connectives

---

Negation (not)	$\neg p$
Conjunction (and)	$p \wedge q$
Disjunction (or)	$p \vee q$
Exclusive Or	$p \oplus q$
Implication	$p \rightarrow q$
Biconditional	$p \leftrightarrow q$

Measles:

“You can get measles”

Mumps:

“You can get mumps”

MMR:

“You had the MMR vaccine”

“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either.”



((Measles and Mumps) if not MMR) and (if MMR then not (Measles or Mumps))



((Measles  $\wedge$  Mumps) if  $\neg$ MMR)  $\wedge$  (if MMR then  $\neg$ (Measles  $\vee$  Mumps))



# Some Truth Tables

---

$p$	$\neg p$

$p$	$q$	$p \wedge q$

$p$	$q$	$p \vee q$

$p$	$q$	$p \oplus q$

# Some Truth Tables

---

$p$	$\neg p$
T	F
F	T

$p$	$q$	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$p$	$q$	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

# Implication

---

*“If it’s raining, then I have my umbrella”*

*It’s useful to think of implications as promises. That is “Did I lie?”*

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

	It’s raining	It’s not raining
I have my umbrella		
I do not have my umbrella		

# Implication

---

*“If it’s raining, then I have my umbrella”*

*It’s useful to think of implications as promises. That is “Did I lie?”*

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

	It’s raining	It’s not raining
I have my umbrella	No	No
I do not have my umbrella	<b>Yes</b>	No

*The only lie is when:*

*(a) It’s raining AND*

*(b) I don’t have my umbrella*

# Implication

---

*“If it’s raining, then I have my umbrella”*

*Are these true?*

$2 + 2 = 4 \rightarrow$  *earth is a planet*

$2 + 2 = 5 \rightarrow$  *26 is prime*

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

# Implication

---

*“If it’s raining, then I have my umbrella”*

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

*Are these true?*

$2 + 2 = 4 \rightarrow$  *earth is a planet*

The fact that these are unrelated doesn’t make the statement false! “ $2 + 2 = 4$ ” is true; “earth is a planet” is true.  $T \rightarrow T$  is true. So, the statement is true.

$2 + 2 = 5 \rightarrow$  *26 is prime*

Again, these statements may or may not be related. “ $2 + 2 = 5$ ” is false; so, the implication is true. (Whether 26 is prime or not is irrelevant).

*Implication is not a causal relationship!*

$$p \rightarrow q$$

---

**(1) “I have collected all 151 Pokémon if I am a Pokémon master”**

**(2) “I have collected all 151 Pokémon only if I am a Pokémon master”**

**These sentences are implications in opposite directions:**

$$p \rightarrow q$$

---

(1) *“I have collected all 151 Pokémon if I am a Pokémon master”*

(2) *“I have collected all 151 Pokémon only if I am a Pokémon master”*

These sentences are implications in opposite directions:

(1) **“Pokémon masters have all 151 Pokémon”**

(2) **“People who have 151 Pokémon are Pokémon masters”**

So, the implications are:

(1) *If I am a Pokémon master, then I have collected all 151 Pokémon.*

(2) *If I have collected all 151 Pokémon, then I am a Pokémon master.*



$$p \rightarrow q$$

---

## Implication:

- $p$  implies  $q$
- whenever  $p$  is true  $q$  must be true
- if  $p$  then  $q$
- $q$  if  $p$
- $p$  is sufficient for  $q$
- $p$  only if  $q$
- $q$  is necessary for  $p$

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

## Biconditional: $p \leftrightarrow q$

---

- $p$  iff  $q$
- $p$  is equivalent to  $q$
- $p$  implies  $q$  and  $q$  implies  $p$
- $p$  is necessary and sufficient for  $q$

$p$	$q$	$p \leftrightarrow q$

## Back to our Vaccine Sentence Translation...

---

**“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine you can’t get either.”**



$((\text{Measles} \wedge \text{Mumps}) \text{ if } \neg \text{MMR}) \wedge (\text{if MMR then } \neg(\text{Measles} \vee \text{Mumps}))$



$(\neg \text{MMR} \rightarrow (\text{Measles} \wedge \text{Mumps})) \wedge (\text{MMR} \rightarrow \neg(\text{Measles} \vee \text{Mumps}))$

# Understanding the Vaccine Sentence

---

“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine you can’t get either.”

$((\text{Measles} \wedge \text{Mumps}) \text{ if } \neg \text{MMR}) \wedge (\text{if MMR then } \neg(\text{Measles} \vee \text{Mumps}))$

$(\neg \text{MMR} \rightarrow (\text{Measles} \wedge \text{Mumps})) \wedge (\text{MMR} \rightarrow \neg(\text{Measles} \vee \text{Mumps}))$

Define shorthand ...

$p$  : MMR

$q$  : Measles

$r$  : Mumps

$(\neg p \rightarrow (q \wedge r)) \wedge (p \rightarrow \neg(q \vee r))$

# Analyzing the Vaccine Sentence with a Truth Table

---

$p$	$q$	$r$	$\neg p$	$q \wedge r$	$\neg p \rightarrow (q \wedge r)$	$q \vee r$	$\neg(q \vee r)$	$p \rightarrow \neg(q \vee r)$	$(\neg p \rightarrow (q \wedge r)) \wedge$ $(p \rightarrow \neg(q \vee r))$
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

# Analyzing the Vaccine Sentence with a Truth Table

---

$p$	$q$	$r$	$\neg p$	$q \wedge r$	$\neg p \rightarrow (q \wedge r)$	$q \vee r$	$\neg(q \vee r)$	$p \rightarrow \neg(q \vee r)$	$(\neg p \rightarrow (q \wedge r)) \wedge (p \rightarrow \neg(q \vee r))$
T	T	T	F	T	T	T	F	F	F
T	T	F	F	F	T	T	F	F	F
T	F	T	F	F	T	T	F	F	F
T	F	F	F	F	T	F	T	T	T
F	T	T	T	T	T	T	F	T	T
F	T	F	T	F	F	T	F	T	F
F	F	T	T	F	F	T	F	T	F
F	F	F	T	F	F	F	T	T	F

# Biconditional: $p \leftrightarrow q$

---

- $p$  iff  $q$
- $p$  is equivalent to  $q$
- $p$  implies  $q$  and  $q$  implies  $p$
- $p$  is necessary and sufficient for  $q$

$p$	$q$	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

# Converse, Contrapositive

---

**Implication:**

$$p \rightarrow q$$

**Converse:**

$$q \rightarrow p$$

**Contrapositive:**

$$\neg q \rightarrow \neg p$$

**Inverse:**

$$\neg p \rightarrow \neg q$$

Consider

$p$ :  $x$  is divisible by 2

$q$ :  $x$  is divisible by 4

$p \rightarrow q$	
$q \rightarrow p$	
$\neg q \rightarrow \neg p$	
$\neg p \rightarrow \neg q$	



# Converse, Contrapositive

---

**Implication:**

$$p \rightarrow q$$

**Converse:**

$$q \rightarrow p$$

**Contrapositive:**

$$\neg q \rightarrow \neg p$$

**Inverse:**

$$\neg p \rightarrow \neg q$$

Consider

**$p$ :  $x$  is divisible by 2**

**$q$ :  $x$  is divisible by 4**

$p \rightarrow q$	
$q \rightarrow p$	
$\neg q \rightarrow \neg p$	
$\neg p \rightarrow \neg q$	

	Divisible By 2	Not Divisible By 2
Divisible By 4		
Not Divisible By 4		

# Converse, Contrapositive

---

**Implication:**

$$p \rightarrow q$$

**Converse:**

$$q \rightarrow p$$

**Contrapositive:**

$$\neg q \rightarrow \neg p$$

**Inverse:**

$$\neg p \rightarrow \neg q$$

Consider

**$p$ :  $x$  is divisible by 2**

**$q$ :  $x$  is divisible by 4**

$p \rightarrow q$	
$q \rightarrow p$	
$\neg q \rightarrow \neg p$	
$\neg p \rightarrow \neg q$	

	Divisible By 2	Not Divisible By 2
Divisible By 4	4,8,12,...	Impossible
Not Divisible By 4	2,6,10,...	1,3,5,...

# Converse, Contrapositive

---

Implication:

$$p \rightarrow q$$

Converse:

$$q \rightarrow p$$

Contrapositive:

$$\neg q \rightarrow \neg p$$

Inverse:

$$\neg p \rightarrow \neg q$$

How do these relate to each other?

$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T	T					
T	F	F					
F	T	T					
F	F	T					

# Converse, Contrapositive

---

Implication:

$$p \rightarrow q$$

Converse:

$$q \rightarrow p$$

Contrapositive:

$$\neg q \rightarrow \neg p$$

Inverse:

$$\neg p \rightarrow \neg q$$

An **implication** and its **contrapositive**  
have the same truth value!

$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T