# CSE 311: Foundations of Computing I

## Section : Strong and Structural Induction

## 1. Strong Induction

(a) Prove that, for all $n \in \mathbb{N}$, every $n$ has an unsigned binary representation.

(b) Xavier Cantelli owns some rabbits. The number of rabbits he has in any given year is described by the function $f$:

$$
\begin{aligned}
f(0) &= 0 \\
f(1) &= 1 \\
f(n) &= 2f(n-1) - f(n-2) \qquad \text{for } n \geq 2
\end{aligned}
$$

Determine, with proof, the number, $f(n)$, of rabbits that Cantelli owns in year $n$.

## 2. Structural Induction

(a) Consider the following recursive definition of strings.

**Basis Step:** `""` is a string

**Recursive Step:** If $X$ is a string and $c$ is a character then append$(c, X)$ is a string.

Recall the following recursive definition of the function len:

$$
\begin{aligned}
\text{len}(\texttt{""}) &= 0 \\
\text{len}(\text{append}(c, X)) &= 1 + \text{len}(X)
\end{aligned}
$$

Now, consider the following recursive definition:

$$
\begin{aligned}
\text{double}(\texttt{""}) &= \texttt{""} \\
\text{double}(\text{append}(c, X)) &= \text{append}(c, \text{append}(c, \text{double}(X))).
\end{aligned}
$$

Prove that for any string $X$, $\text{len}(\text{double}(X)) = 2\text{len}(X)$.

(b) Consider the following definition of a (binary) **Tree**:

**Basis Step:** $\bullet$ is a **Tree**.

**Recursive Step:** If $L$ is a **Tree** and $R$ is a **Tree** then $\texttt{Tree}(\bullet, L, R)$ is a **Tree**.

The function leaves returns the number of leaves of a **Tree**. It is defined as follows:

$$
\begin{aligned}
\text{leaves}(\bullet) &= 1 \\
\text{leaves}(\texttt{Tree}(\bullet, L, R)) &= \text{leaves}(L) + \text{leaves}(R)
\end{aligned}
$$

Also, recall the definition of size on trees:

$$
\begin{aligned}
\text{size}(\bullet) &= 1 \\
\text{size}(\texttt{Tree}(\bullet, L, R)) &= 1 + \text{size}(L) + \text{size}(R)
\end{aligned}
$$

Prove that $\text{leaves}(T) \geq \text{size}(T)/2$ for all Trees $T$.