CSE 311: Foundations of Computing I

Section : English Proofs, Sets, and Modular Arithmetic

1. Odds and Ends

Prove that for any even integer, there exists an odd integer greater than that even integer.

2. Primality Checking

When brute forcing if the number n is prime, you only need to check possible factors up to $\sqrt{n}$. In this problem, you'll prove why that is the case. Prove that if $n = ab$, then either a or b is at most $\sqrt{n}$.

(*Hint:* You want to prove an implication; so, start by assuming $n = ab$. Then, continue by writing out your assumption for contradiction.)

3. How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say .

(a) $A = \{1, 2, 3, 2\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \ldots\}$

(c) $C = A \times (B \cup \{7\})$

(d) $D = \varnothing$

(e) $E = \{\varnothing\}$

(f) $F = \mathcal{P}(\{\varnothing\})$

4. Set = Set

Prove the following set identities.

(a) Let the universal set be $\mathcal{U}$. Prove $\overline{\overline{X}} = X$ for any set X.

(b) Prove $(A \oplus B) \oplus B = A$ for any sets A, B.

(c) Prove $A \cup B \subseteq A \cup B \cup C$ for any sets A, B, C.

(d) Let the universal set be $\mathcal{U}$. Prove $A \cap \overline{B} \subseteq A \setminus B$ for any sets A, B.

5. Modular Arithmetic

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

(b) Prove that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{n}$.