CSE 311: Foundations of Computing I

Section 3: Predicate Logic and Inference

1. Translate to Logic

Express each of these system specifications using predicate, quantifiers, and logical connectives.

(a) Every user has access to an electronic mailbox.

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

(c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

(d) At least one router is functioning normally if the throughput is between 100kbps and 500 kbps and the proxy server is not in diagnostic mode.

2. Translate to English

Translate these system specifications into English where $F(p)$ is "Printer p is out of service", $B(p)$ is "Printer p is busy", $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued". Let the domain be all printers together with all print jobs.

(a) $\exists p\,(F(p) \wedge B(p)) \rightarrow \exists j\, L(j)$

(b) $(\forall p\, B(p)) \rightarrow (\exists j\, Q(j))$

(c) $\exists j\,(Q(j) \wedge L(j)) \rightarrow \exists p\, F(p)$

(d) $(\forall p\, B(p) \wedge \forall j\, Q(j)) \rightarrow \exists j\, L(j)$

3. Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other (or if they are equivalent).

(a) $\forall x\, \forall y\, P(x, y)$ $\quad$ $\forall y\, \forall x\, P(x, y)$

(b) $\exists x\, \exists y\, P(x, y)$ $\quad$ $\exists y\, \exists x\, P(x, y)$

(c) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\forall y\, \exists x\, P(x, y)$

(d) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\exists x\, \forall y\, P(x, y)$

4. Formal Proof (Direct Proof Rule)

Show that $\neg p \rightarrow s$ follows from $p \vee q$, $q \rightarrow r$ and $r \rightarrow s$.

5. Formal Proof

Show that $\neg p$ follows from $\neg(\neg r \vee t)$, $\neg q \vee \neg s$ and $(p \rightarrow q) \wedge (r \rightarrow s)$.