CSE 311: Foundations of Computing I

Homework 9 (due Friday, Dec 9nd at 5:00 PM)

1. Pattern Matching [Online] (15 points)

Use the method given in class to design a DFA to determine all occurrences of the string 11011011001 in strings over the alphabet $\{0, 1\}$.

You must submit and check your answers to this question using
https://grinch.cs.washington.edu/cse311/fsm.

2. Diagonalization (20 points)

Let B be the set of all infinite binary sequences that are 1 in odd positions, i.e., any string in B is of the form

$$1 * 1 * 1 * 1 * \ldots$$

where we can have 0 or a 1 instead of each $*$. Show that B is uncountable using a proof by diagonalization.

3. Countability (20 points)

Complex numbers can be written as $a + bi$ where a, b are real numbers and i is the square root of -1. Show that subset R of complex numbers given by

$$R = \{a + bi : a, b \text{ are rational}\}$$

is countable

4. Irregularity (30 points)

Using the method shown in class prove that that the following languages are not regular.

(a) [15 Points] The set of binary strings of the form $\{0^n 1^m 0^n : m < n\}$.

(b) [15 Points] The set of strings 0^n where n is a perfect square, i.e., $n = k^2$ for some $k \in \mathbb{N}$.

5. Undecidability (15 points)

Consider the set

$$\mathbf{Prime} = \{(\text{CODE}(\mathbf{P}), \mathbf{x}) : \mathbf{P} \text{ reads } \mathbf{x} \text{ and halts if and only if } \mathbf{x} \text{ is a prime}\}$$

Show that $\mathbf{Prime}$ is undecidable using the fact that the Halting Problem is undecidable.