cse 311: foundations of computing

Spring 2015

Lecture 25: Limitations of DFAs (irregular languages)

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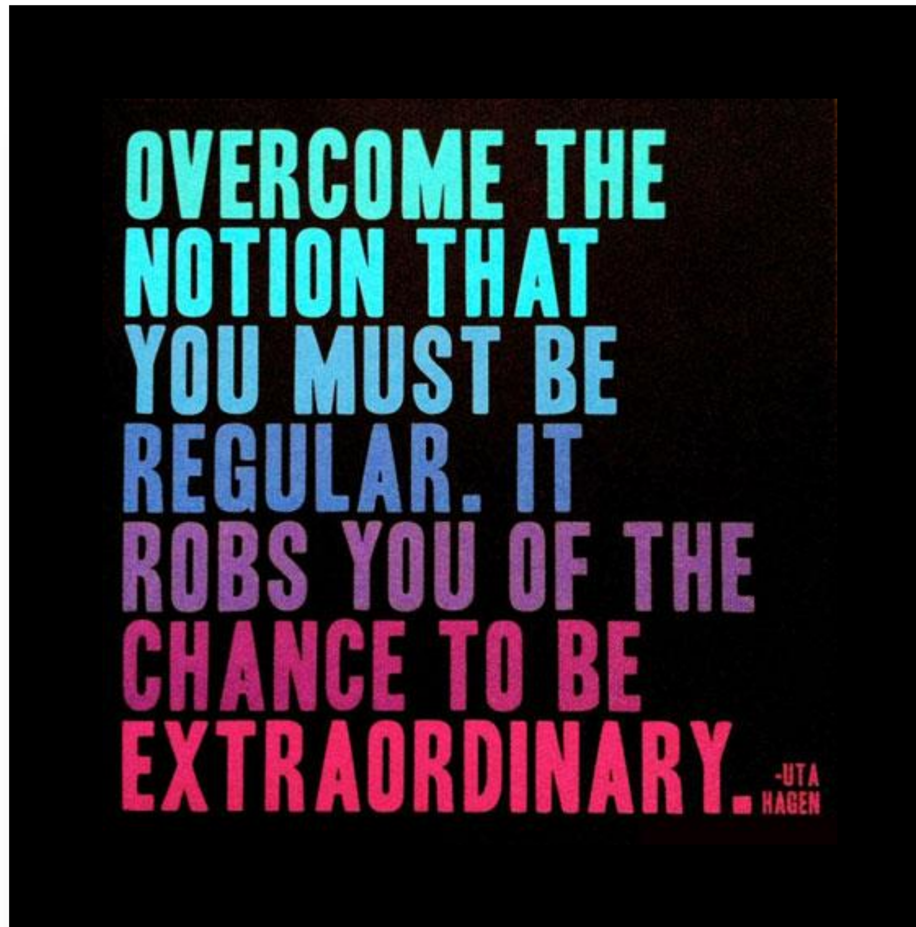


B = {binary palindromes} can’t be recognized by any DFA

Why is this language not regular?

Intuition (**not a proof**):

Q: What would a DFA need to keep track of to decide the language?
A: It would need to keep track of the “first part” of the input in order to check the second part against it… but there are an infinite # of possible first parts and we only have finitely many states.

How do we prove it?

B = {binary palindromes} can't be recognized by any DFA

Consider the infinite set of strings

$$S = \{1, 01, 001, 0001, 00001, \ldots\} = \{0^n 1 : n \geq 0\}$$

That's a nice set of first parts to have to remember but how can we argue that a DFA does the wrong thing for B?

- Show that some **x** ∈ B and some y ∉ B both must end up at the *same* state of the DFA

That state can't be

- a final state since then y is accepted: error on y
- a non-final state since then x is rejected: error on x

B = {binary palindromes} can't be recognized by any DFA

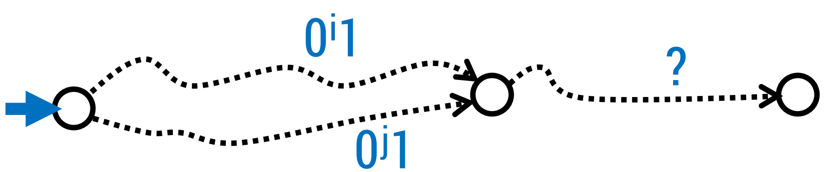
Consider the infinite set of strings

$$S = \{1, 01, 001, 0001, 00001, \ldots\} = \{0^n 1 : n \geq 0\}$$

Suppose we are given an arbitrary DFA **M**.

- Goal: Show that some **x** ∈ **B** and some **y** ∉ **B** both must end up at the *same* state of **M**

Since **S** is infinite we know that two different strings in **S** must land in the same state of **M**, call them $\mathbf{0^i1}$ and $\mathbf{0^j1}$ for i≠j.



- That also must be true for $\mathbf{0^i1z}$ and $\mathbf{0^j1z}$ for any **z** ∈ $\{\mathbf{0,1}\}^*$ **!**

In particular, with $\mathbf{z=0^i}$ we get that $\mathbf{0^i10^i}$ and $\mathbf{0^j10^i}$ end up at the same state of **M**. Since $\mathbf{0^i10^i}$ ∈ **B** and $\mathbf{0^j10^i}$ ∉ **B** (because i≠j), M does not recognize **B**.
∴ no DFA can recognize **B**.

showing a language L is not regular

1. Find an infinite set **S**={$\mathbf{s_0}$,$\mathbf{s_1}$,…,$\mathbf{s_n}$,…} of string prefixes that you think will need to be remembered separately

2. "Let **M** be an arbitrary DFA. Since **S** is infinite and **M** is finite state there must be two strings $\mathbf{s_i}$ and $\mathbf{s_j}$ in **S** for some **i** ≠ **j** that end up at the same state of **M**."

   Note: You don't get to choose which two strings $\mathbf{s_i}$ and $\mathbf{s_j}$

3. Find a string **t** (typically depending on $\mathbf{s_i}$ and/or $\mathbf{s_j}$) such that

   $\mathbf{s_i t}$ is in **L**, and $\mathbf{s_j t}$ is not in **L** or $\mathbf{s_i t}$ is not in **L**, and $\mathbf{s_j t}$ is in **L**

4. "Since $\mathbf{s_i}$ and $\mathbf{s_j}$ both end up at the same state of **M**, and we appended the same string **t**, both $\mathbf{s_i t}$ and $\mathbf{s_j t}$ end at the same state of **M.** Since $\mathbf{s_i t} \in \mathbf{L}$ and $\mathbf{s_j t} \notin \mathbf{L}$, **M** does not recognize **L**."

5. "Since **M** was arbitrary, no DFA recognizes **L**."

$A = \{0^n 1^n : n \geq 0\}$ cannot be recognized by any DFA

1. Find an infinite set **S**=$\{s_0, s_1, \ldots, s_n, \ldots\}$ of string prefixes that you think will need to be remembered separately

2. "Let **M** be an arbitrary DFA. Since **S** is infinite and **M** is finite state there must be two strings s_i and s_j in **S** for some $i \neq j$ that end up at the same state of **M**."

3. Find a string **t** (typically depending on s_i and/or s_j) such that
   $s_i t$ is in **L**, and
   $s_j t$ is not in **L**

4. "Since s_i and s_j both end up at the same state of **M**, and we appended the same string **t**, both $s_i t$ and $s_j t$ end at the same state of **M**. Since $s_i t \in L$ and $s_j t \notin L$, **M** does not recognize **L**."

5. "Since **M** was arbitrary, no DFA recognizes **L**."

$A = \{0^n 1^n : n \geq 0\}$ cannot be recognized by any DFA

1. Find an infinite set **S**=$\{s_0, s_1, \ldots, s_n, \ldots\}$ of string prefixes that you think will need to be remembered separately $S = \{0^n : n \geq 0\}$

2. "Let **M** be an arbitrary DFA. Since **S** is infinite and **M** is finite state there must be two strings s_i and s_j in **S** for some **i** ≠ **j** that end up at the same state of **M**."

   s_i, s_j

3. Find a string **t** (typically depending on s_i and/or s_j) such that

   $s_i t$ is in **L**, and

   $s_j t$ is not in **L**

   $t = 1^i$

4. "Since s_i and s_j both end up at the same state of **M**, and we appended the same string **t**, both $s_i t$ and $s_j t$ end at the same state of **M.** Since $s_i t \in L$ and $s_j t \notin L$, **M** does not recognize **L**."

5. "Since **M** was arbitrary, no DFA recognizes **L**."

another irregular language

L = {$x \in \{0, 1,2\}^*$: x has an equal number of substrings 01 and 10}.

Intuition: Need to remember difference in # of **01** or **10** substrings seen, but only hard to do if these are separated by **2**'s.

1. Let **S**={ε, 012, 012012, 012012012, ...} = $\{(012)^n : n \in \mathbb{N}\}$

2. Let **M** be an arbitrary DFA. Since **S** is infinite and **M** is finite, there must be two strings $(012)^i$ and $(012)^j$ for some $i \neq j$ that end up at the same state of **M**.

3. Consider appending string **t** = $(102)^i$ **to each of these strings.**

   Then $(012)^i (102)^i \in$ **L** but $(012)^j (102)^i \notin$ **L** since $i \neq j$

4. So $(012)^i (102)^i$ and $(012)^j (102)^i$ end up at the same state of **M** since $(012)^i$ and $(012)^j$ do. Since $(012)^i (102)^i \in$ **L** and $(012)^j (102)^i \notin$ **L**, **M** does not recognize **L**.

5. Since **M** was arbitrary, no DFA recognizes **L**.

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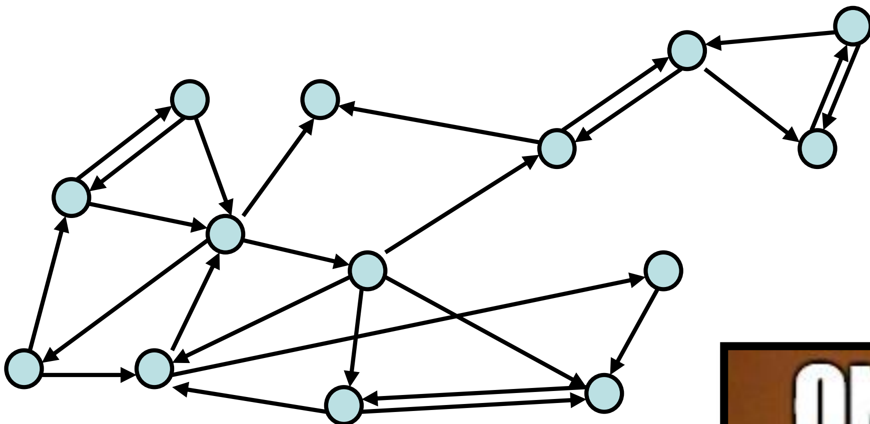
Lecture 26: Relations and directed graphs



Let A and B be sets.
A binary relation from A to B is a subset of $A \times B$

Let A be a set.
A binary relation on A is a subset of $A \times A$

relations you already know!

$\geq$ on $\mathbb{N}$

That is: $\{(x,y) : x \geq y \text{ and } x, y \in \mathbb{N}\}$

$<$ on $\mathbb{R}$

That is: $\{(x,y) : x < y \text{ and } x, y \in \mathbb{R}\}$

$=$ on Σ^*

That is: $\{(x,y) : x = y \text{ and } x, y \in \Sigma^*\}$

$\subseteq$ on $P(U)$ for universe U

That is: $\{(A,B) : A \subseteq B \text{ and } A, B \in P(U)\}$

examples

$R_1 = \{(a, 1), (a, 2), (b, 1), (b, 3), (c, 3)\}$

$R_2 = \{(x, y) \mid x \equiv y \pmod 5\}$

$R_3 = \{(c_1, c_2) \mid c_1 \text{ is a prerequisite of } c_2\}$

$R_4 = \{(s, c) \mid \text{student } s \text{ had taken course } c\}$

Let R be a relation on A.

R is reflexive iff $(a,a) \in R$ for every $a \in A$

R is symmetric iff $(a,b) \in R$ implies $(b, a) \in R$

R is antisymmetric iff $(a,b) \in R$ and $a \neq b$ implies $(b,a) \notin R$

R is transitive iff $(a,b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

combining relations

Let R be a relation from A to B.
Let S be a relation from B to C.

The composition of R and S, $S \circ R$ is the relation from A to C defined by:

$$S \circ R = \{(a, c) \mid \exists\, b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$$

Intuitively, a pair is in the composition if there is a "connection" from the first to the second.

$(a,b) \in$ Parent iff b is a parent of a

$(a,b) \in$ Sister iff b is a sister of a

When is $(x,y) \in$ Sister $\circ$ Parent?

When is $(x,y) \in$ Parent $\circ$ Sister?

$S \circ R = \{(a, c) \mid \exists\, b$ such that $(a,b) \in R$ and $(b,c) \in S\}$

examples

Using the relations: Parent, Child, Brother, Sister, Sibling, Father, Mother, Husband, Wife express:

Uncle: b is an uncle of a

Cousin: b is a cousin of a

Using the relations: Parent, Child, Brother, Sister, Sibling, Father, Mother, Husband, Wife express:

Uncle: b is an uncle of a

$$\text{Uncle} = \text{Brother} \circ \text{Parent}$$

Cousin: b is a cousin of a

$$\text{Cousin} = \text{Child} \circ \text{Sibling} \circ \text{Parent}$$

powers of a relation

$$R^2 = R \circ R$$
$$= \{(a, c) \mid \exists b \text{ such that } (a, b) \in R \text{ and } (b, c) \in R\}$$

$$R^0 = \{(a, a) \mid a \in A\}$$

$$R^1 = R$$

$$R^{n+1} = R^n \circ R$$

powers of a relation

$$R^2 = R \circ R$$
$$= \{(a, c) \mid \exists b \text{ such that } (a, b) \in R \text{ and } (b, c) \in R\}$$

$$\text{Parent}^2 = \text{GrandParent}$$

$$R^0 = \{(a, a) \mid a \in A\}$$

R^0 is always equality

$$R^1 = R$$

$$R^{n+1} = R^n \circ R$$

matrix represenation

Relation R **on** $A = \{a_1, \ldots, a_p\}$

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, a_j) \in R, \\ 0 \text{ if } (a_i, a_j) \notin R. \end{cases}$$

{(1, 1), (1, 2), (1, 4), (2,1), (2,3), (3,2), (3, 3), (4,2), (4,3)}

	1	2	3	4
1	1	1	0	1
2	1	0	1	0
3	0	1	1	0
4	0	1	1	0