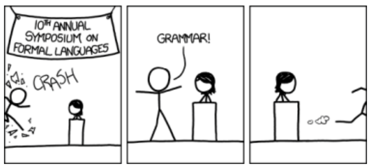
cse 311: foundations of computing

Spring 2015
Lecture 20: Regular expressions and context-free grammars



languages: sets of strings

Sets of strings that satisfy special properties are called languages.

Examples:
- English sentences
- Syntactically correct Java/C/C++ programs
- Σ^* = All strings over alphabet Σ
- Palindromes over Σ
- Binary strings that don't have a 0 after a 1
- Legal variable names, keywords in Java/C/C++
- Binary strings with an equal # of 0's and 1's

regular expressions

Regular expressions over Σ

- Basis:
  - $\varnothing$, ε are regular expressions
  - a is a regular expression for any $a \in \Sigma$
- Recursive step:
  - If **A** and **B** are regular expressions then so are:
    - **(A $\cup$ B)**
    - **(AB)**
    - **A***

each regular expression is a "pattern"

ε matches the empty string

a matches the one character string a

(A $\cup$ B) matches all strings that either **A** matches or **B** matches (or both)

(AB) matches all strings that have a first part that **A** matches followed by a second part that **B** matches

A* matches all strings that have any number of strings (even 0) that **A** matches, one after another

examples

- 001*
- 0*1*
- $(0 \cup 1)0(0 \cup 1)0$
- (0*1*)*
- $(0 \cup 1)^*$ 0110 $(0 \cup 1)^*$
- $(00 \cup 11)^*$ $(01010 \cup 10001)(0 \cup 1)^*$

regular expressions in practice

- Used to define the "tokens": e.g., legal variable names, keywords in programming languages and compilers
- Used in **grep**, a program that does pattern matching searches in UNIX/LINUX
- Pattern matching using regular expressions is an essential feature of PHP
- We can use regular expressions in programs to process strings!

regular expressions in Java

- Pattern p = Pattern.compile("a*b");
- Matcher m = p.matcher("aaaaab");
- boolean b = m.matches();

  `[01]` a 0 or a 1 `^` start of string `$` end of string

  `[0-9]` any single digit `\.` period `\,` comma `\-` minus

  `.` any single character

ab	a followed by b	**(AB)**
(a | b)	a or b	$(\mathbf{A} \cup \mathbf{B})$
a?	zero or one of a	$(\mathbf{A} \cup \varepsilon)$
a*	zero or more of a	**A***
a+	one or more of a	**AA***

- e.g. `^[\-+]?[0-9]*(\.|\,)?[0-9]+$`

  General form of decimal number e.g. 9.12 or -9,8 (Europe)

matching email addresses: RFC 822

more examples

- All binary strings that have an even # of 1's
- All binary strings that *don't* contain 101

limitations of regular expressions

- Not all languages can be specified by regular expressions
- Even some easy things like
  - Palindromes
  - Strings with equal number of 0's and 1's
- But also more complicated structures in programming languages
  - Matched parentheses
  - Properly formed arithmetic expressions
  - etc.

context-free grammars

- A Context-Free Grammar (CFG) is given by a finite set of substitution rules involving
  - A finite set **V** of *variables* that can be replaced
  - Alphabet Σ of *terminal symbols* that can't be replaced
  - One variable, usually **S**, is called the *start symbol*
- The rules involving a variable **A** are written as

  $\mathbf{A} \rightarrow w_1 \mid w_2 \mid \cdots \mid w_k$

  where each w_i is a string of variables and terminals:

  $w_i \in (\mathbf{V} \cup \Sigma)^*$

how CFGs generate strings

- Begin with start symbol **S**
- If there is some variable **A** in the current string you can replace it by one of the w's in the rules for **A**
  - $\mathbf{A} \rightarrow w_1 \mid w_2 \mid \cdots \mid w_k$
  - Write this as $x\mathbf{A}y \Rightarrow xwy$
  - Repeat until no variables left
- The set of strings the CFG generates are all strings produced in this way that have no variables

example

Example: S → 0S0 | 1S1 | 0 | 1 | ε

Example: S → 0S | S1 | ε

example

Grammar for $\{0^n 1^n : n \geq 0\}$
(all strings with same # of 0's and 1's with all 0's before 1's)

Example: S → (S) | SS | ε

simple arithmetic expressions

E → E+E | E∗E | (E) | x | y | z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Generate (2∗x) + y

Generate x+y∗z in two fundamentally different ways

parse trees

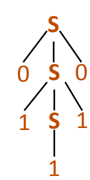
Suppose that grammar G generates a string x

A **parse tree** of x for G has

- Root labeled S (start symbol of G)
- The children of any node labeled A are labeled by symbols of w left-to-right for some rule A → w
- The symbols of x label the leaves ordered left-to-right

S → 0S0 | 1S1 | 0 | 1 | ε

Parse tree of 01110:



CFGs and recursively-defined sets of strings

- A CFG with the start symbol **S** as its only variable recursively defines the set of strings of terminals that **S** can generate
- A CFG with more than one variable is a simultaneous recursive definition of the sets of strings generated by *each* of its variables
  - Sometimes necessary to use more than one

building precedence in simple arithmetic expressions

- **E** – expression (start symbol)
- **T** – term **F** – factor **I** – identifier **N** - number

E → **T** | **E**+**T**
T → **F** | **F**∗**T**
F → (**E**) | **I** | **N**
I → x | y | z
N → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Backus-Naur form (same as CFG)

BNF (Backus-Naur Form) grammars

- Originally used to define programming languages
- Variables denoted by long names in angle brackets, e.g.
  <identifier>, <if-then-else-statement>,
  <assignment-statement>, <condition>
  ::= used instead of →

BNF for C

```
statement:
  ((identifier | "case" constant-expression | "default") ":")*
  (expression? ";" |
   block |
   "if" "(" expression ")" statement |
   "if" "(" expression ")" statement "else" statement |
   "switch" "(" expression ")" statement |
   "while" "(" expression ")" statement |
   "do" statement "while" "(" expression ")" ";" |
   "for" "(" expression? ";" expression? ";" expression? ")" statement |
   "goto" identifier ";" |
   "continue" ";" |
   "break" ";" |
   "return" expression? ";"
  )

block: "{" declaration* statement* "}"

expression:
  assignment-expression%

assignment-expression: (
    unary-expression (
      "=" | "*=" | "/=" | "%=" | "+=" | "-=" | "<<=" | ">>=" | "&=" |
      "^=" | "|="
    )
  )* conditional-expression

conditional-expression:
  logical-OR-expression ( "?" expression ":" conditional-expression )?
```

parse trees

Back to middle school:

<sentence>::=<noun phrase><verb phrase>
<noun phrase>::==<article><adjective><noun>
<verb phrase>::=<verb><adverb>|<verb><object>
<object>::=<noun phrase>

Parse:

The yellow duck squeaked loudly
The red truck hit a parked car