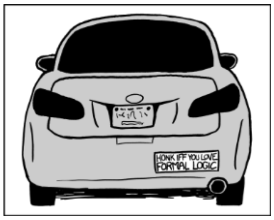
CSE 311: Foundations of Computing I

Spring 2015
Lecture 1: Propositional Logic



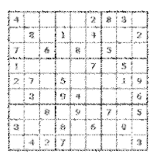
about the course

We will study the **theory** needed for CSE.

Logic:
How can we describe ideas and arguments **precisely**?
Formal proofs:
Can we prove that we're right? [to ourselves? to others?]
Number theory:
How do we keep data **secure**? [really? we need to justify numbers?]
Relations/Relational Algebra:
How do we store information?
How do we reason about the effects of connectivity?
Finite state machines:
How do we design hardware and software? [state!]
Turing machines:
What is computation? [the universe? superheroes?]
Are there problems computers **can't** solve?

about the course

The computational perspective.



Example: Sudoku
Given *one*, solve by hand.
Given *most*, solve with a program.
Given *any*, solve with computer science.

[given one, by hand
given most, with a program
. . . computer science]

- Tools for reasoning about difficult problems
- Tools for communicating ideas, methods, objectives
- Fundamental structures for computer science

[like, uhh, smart stuff]

administrivia

Prof: James R. Lee [James "PG 13" Lee was less fun]

Teaching assistants:
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Quiz Sections:
Thursdays

(Optional) **Book**:
Rosen
Discrete Mathematics
6th or 7th edition
Can buy online for ~$50

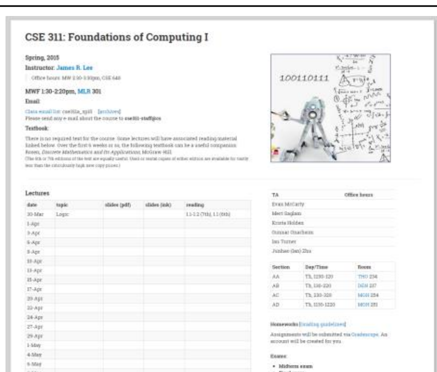
Homework:
Due **Fridays** on **Gradescope**
Write up individually

Exams:
Midterm: date soon
Final: TBA

Grading (roughly):
50% homework
35% final exam
15% midterm

All course information at http://www.cs.washington.edu/311.

administrivia



logic: the language of reasoning

- Why not use English?
  - Turn right here…
  - Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo.

    [The sentence means "Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo."]
  - We saw her duck.
- "Language of Reasoning" like Java or English
  - Words, sentences, paragraphs, arguments…
  - Today is about **words** and **sentences**.

why learn a new language?

Logic as the "language of reasoning", will help us…

- Be more **precise**
- Be more **concise**
- Figure out what a statement means more **quickly**

[please stop]

propositions

A **proposition** is a statement that

- has a truth value, and
- is "well-formed"

["If I were to ask you out, would your answer to that question be the same as your answer to this one?"]

proposition is a statement that has a truth value and is "well-formed"

Consider these statements:

- 2 + 2 = 5
- The home page renders correctly in IE.
- This is the song that never ends.
- Turn in your homework on Wednesday.
- This statement is false.
- Akjsdf? [hey, I akjsdf you a question]
- The Washington State flag is red.
- Every positive even integer can be written as the sum of two primes.

propositions

- A **proposition** is a statement that
  - has a truth value, and
  - is "well-formed"
- Propositional variables: $p, q, r, s, \ldots$
- Truth values: **T** for true, **F** for false

a proposition

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

[might as well just end it all now, Roger]

- What does this proposition mean?
- It seems to be built out of other, more basic propositions that are sitting inside it! What are they?

a proposition

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

RElephant : "Roger is an orange elephant"
RTusks : "Roger has tusks"
RToenails : "Roger has toenails"

logical connectives

- Negation (not) $\neg p$
- Conjunction (and) $p \land q$
- Disjunction (or) $p \lor q$
- Exclusive or $p \oplus q$
- Implication $p \to q$
- Biconditional $p \leftrightarrow q$

RElephant :
"Roger is an orange elephant"
RTusks :
"Roger has tusks"
RToenails :
"Roger has toenails"

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

RElephant **and** (RToenails **if** RTusks) **and** (RToenails **or** RTusks **or** (RToenails **and** RTusks))

some truth tables

p	¬ p

p	q	p ∧ q

p	q	p ∨ q

p	q	p ⊕ q

$p \to q$

"If p, then q" is a **promise**:
- Whenever p is true, then q is true
- Ask "has the promise been broken?"

p	q	$p \to q$

If it's raining, then I have my umbrella.
Suppose it's not raining...

$p \to q$

"I am a Pokémon master only if I have collected all 151 Pokémon."
Can we re-phrase this as "if p, then q" ?

$p \to q$

Implication:
- p implies q
- whenever p is true q must be true
- if p then q
- q if p
- p is sufficient for q
- p only if q

p	q	$p \to q$

converse, contrapositive, inverse

- Implication: $p \to q$
- Converse: $q \to p$
- Contrapositive: $\neg q \to \neg p$
- Inverse: $\neg p \to \neg q$

How do these relate to each other?

back to Roger

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

REIephant ∧ (RToenails **if** RTusks) ∧ (RToenails ∨ RTusks ∨ (RToenails ∧ RTusks))

Define shorthand ...

p : RElephant
q : RTusks
r : RToenails

roger's sentence with a truth table

p	q	r	$q \to r$	$p \wedge (q \to r)$	$r \vee q$	$r \wedge q$	$(r \vee q) \vee (r \wedge q)$	$p \wedge (q \to r) \wedge (r \vee q \vee (r \wedge q))$

Shorthand:

p : RElephant
q : RTusks
r : RToenails

more about roger

Roger is only orange if whenever he either has tusks or toenails, he doesn't have tusks and he is an orange elephant."

p : "Roger is an orange elephant"
q : "Roger has tusks"
r : "Roger has toenails"

more about roger

Roger is only orange if whenever he either has tusks or toenails, he doesn't have tusks and he is an orange elephant."

(RElephant **only if (whenever (**RTusks **xor** RToenails**) then not** RTusks**)) and** RElephant

(RElephant → (whenever **(**RTusks ⊕ RToenails**) then** ¬ RTusks**))** ∧ RElephant

p : RElephant
q : RTusks
r : RToenails

Roger's second sentence with a truth table

p	q	r	$q \oplus r$	$\neg q$	$((q \oplus r) \to \neg q)$	$p \to ((q \oplus r) \to \neg q)$	$(p \to ((q \oplus r) \to \neg q)) \wedge p$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

biconditional: $p \leftrightarrow q$

- p iff q
- p is equivalent to q
- p implies q and q implies p

p	q	$p \leftrightarrow q$