CSE 311: Foundations of Computing (Spring, 2015)

Homework 3 Out: Friday, 17-Apr. Due: Friday, 24-Apr, **before class (1 pm)** on **Gradescope**

Additional directions: You should write down carefully argued solutions to the following problems. Your first goal is to be complete and correct. A secondary goal is to keep your answers simple and succinct. The idea is that your solution should be easy to understand for someone who has just seen the problem for the first time. (Re-read your answers with this standard in mind!) You may use any results proved in lecture (without proof). Anything else must be argued rigorously. A "formal proof" means one where every line is labeled using the logical inference rules and equivalences given in lecture.

1. A formal proof (10 points)

(a) [8 points] Write a formal proof that under the three assumptions:

$$r, \ \ (t \wedge r) \rightarrow s, \ \ (p \rightarrow t) \rightarrow (q \rightarrow \neg s),$$

the proposition $t \rightarrow \neg q$ holds true.

(b) [2 points] How many rows would you need for a truth table proving the same statement?

2. A proof with quantifiers (10 points)

Write a formal proof of that, given

$$\forall x\,\big(\neg P(x) \to Q(x)\big), \qquad \forall x\,\Big(\big(\neg P(x) \land Q(x)\big) \to R(x)\Big), \qquad \neg\forall x\, P(x),$$

you can conclude that $\exists x\, R(x)$.

3. A faulty proof (12 points)

Theorem: Given $(p \wedge r) \rightarrow (q \wedge r)$, $\neg s \rightarrow (r \wedge q)$, and $s \rightarrow (p \wedge q)$, prove q.

"Proof:"

1.	$(p \wedge r) \rightarrow (q \wedge r)$	[Given]
2.	$\neg s \rightarrow (r \wedge q)$	[Given]
3.	$s \rightarrow (p \wedge q)$	[Given]
4.	$\neg s \rightarrow q$	[Elim $\wedge$: 2]
5.	$p \rightarrow (q \wedge r)$	[Elim $\wedge$: 1]
	6. s	[Assumption]
	7. $p \wedge q$	[MP: 6, 3]
	8. p	[Elim $\wedge$: 7]
	9. $q \wedge r$	[MP: 8, 5]
	10. q	[Elim $\wedge$: 9]
11.	$s \rightarrow q$	[Direct Proof Rule]
12.	$(s \rightarrow q) \wedge (\neg s \rightarrow q)$	[Intro $\wedge$: 4, 11]
13.	$(\neg s \vee q) \wedge (\neg\neg s \vee q)$	[Law of Implication]
14.	$(q \vee \neg s) \wedge (q \vee \neg\neg s)$	[Commutativity]
15.	$q \wedge (\neg s \vee \neg\neg s)$	[Distributivity]
16.	$q \wedge T$	[Law of Negation]
17.	q	[Domination]

(a) [3 points] Is the conclusion of the “proof” correct? Explain in words why or why not.

(b) [9 points] Explain which steps of the proof are faulty and why.

4. Rationality (12 points)

A real number x is **rational** if and only if there exist integers p and $q \neq 0$ such that $x = p/q$. In other words,

$$\text{Rational}(x) = \exists p\, \exists q \left(\left(x = \frac{p}{q} \right) \wedge \text{Integer}(p) \wedge \text{Integer}(q) \wedge q \neq 0 \right)$$

For the next two parts, first write the statement in formal logic. Then give a proof of the statement in English (or formal logic, if you wish).

(a) If x and y are rational, then $x + y$ is rational.

(b) If x is a non-zero rational, then $1/x$ is rational.

5. Elementary number theory (12 points)

For each of the following, give either a formal proof or an English proof.

(a) Prove or disprove: For every intenger n, if $5n + 3$ is odd, then n is even. You should use the definitions of even and odd given in class. You can use the property (shown in class) that no number is both even and odd. You can also use the fact that every number is either even or odd.

(b) Prove or disprove: For every integer $n \geq 0$, the expression $n^2 + n + 41$ is prime.

6. Extra credit: Divisibility by 3? Nah, try 11 and 13.

(a) Prove that a number is divisible by 11 if and only if the difference between the sum of its decimal digits in the even positions the sum of its decimal digits in the odd positions is divisible by 11.

(b) Prove that a number is divisible by 13 if and only if the following outputs "TRUE"

1) Write the number in decimal, e.g. 9,527,688,677.
2) Form a sum of the blocks of three digits with alternating signs, e.g. -9+527-688+677 = 507
3) Add 4 times the last digit to the number in (2) without the last digit: 50 + 4*7 = 78
4) Subtact 9 times the last digit from the number in (2) without the last digit: 50 – 7*9 = -13
5) If the results of (3) and (4) are both divisible by 13, then output "TRUE."
6) Else output "FALSE."