

cse 311: foundations of computing

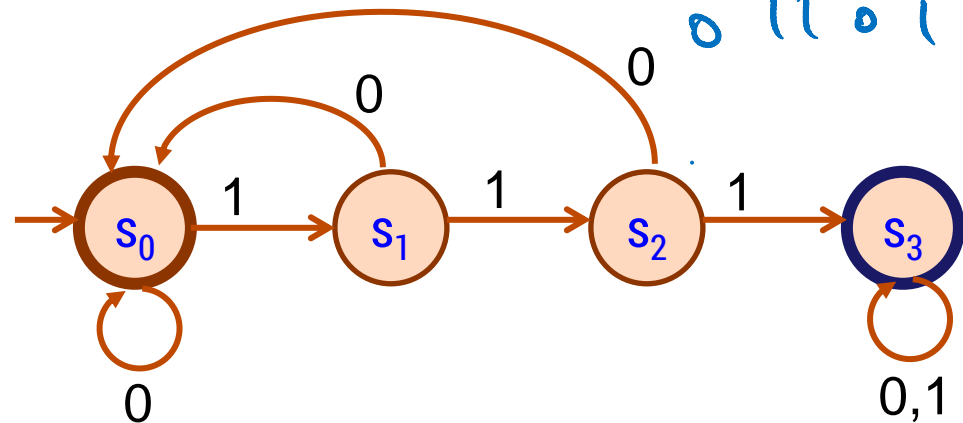
Fall 2015

Lecture 22: Finite state machines

review: finite state machines

- States
- Transitions on inputs
- Start state and final states
- The language recognized by a machine is the set of strings that reach a final state

State	0	1
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_3
s_3	s_3	s_3

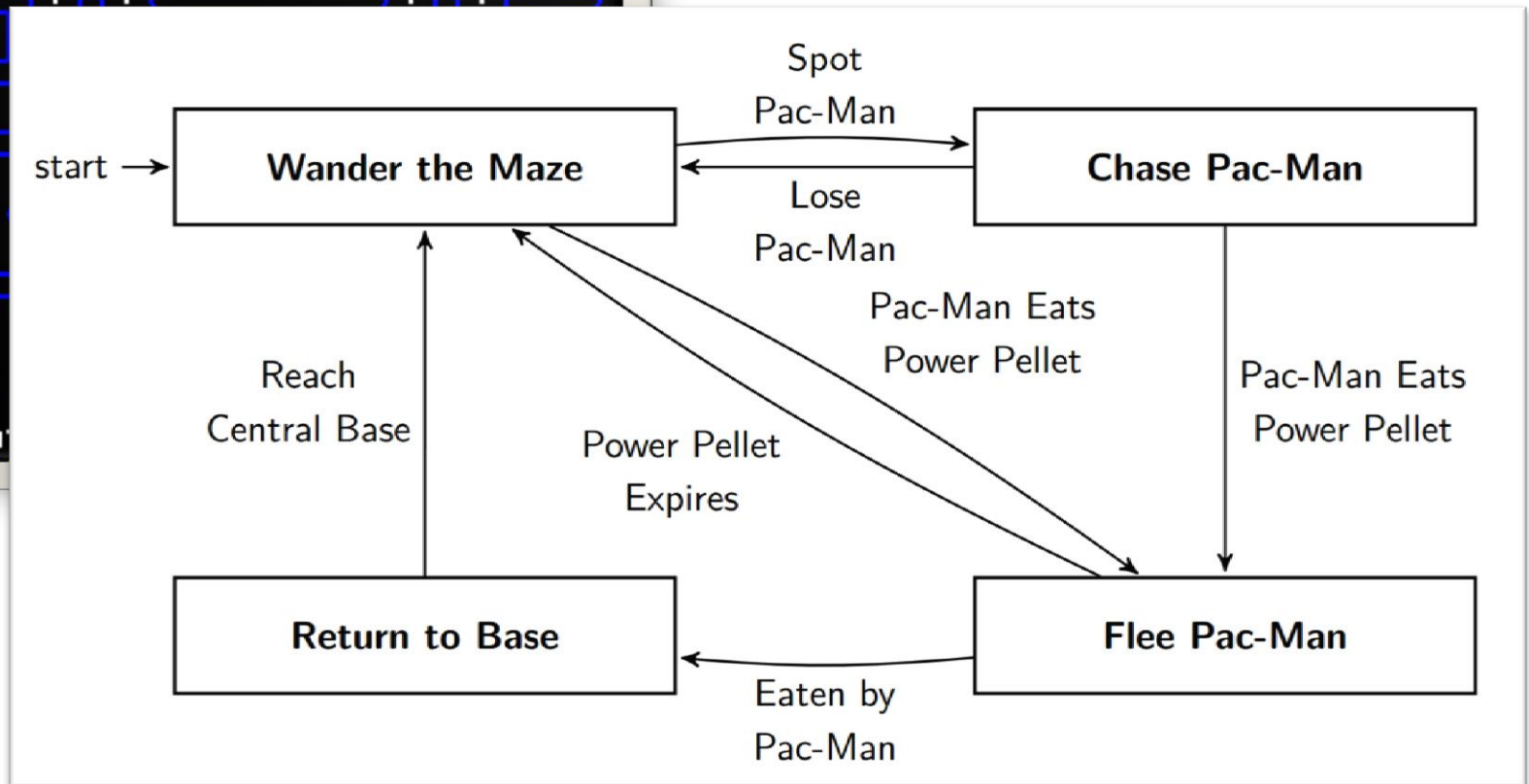
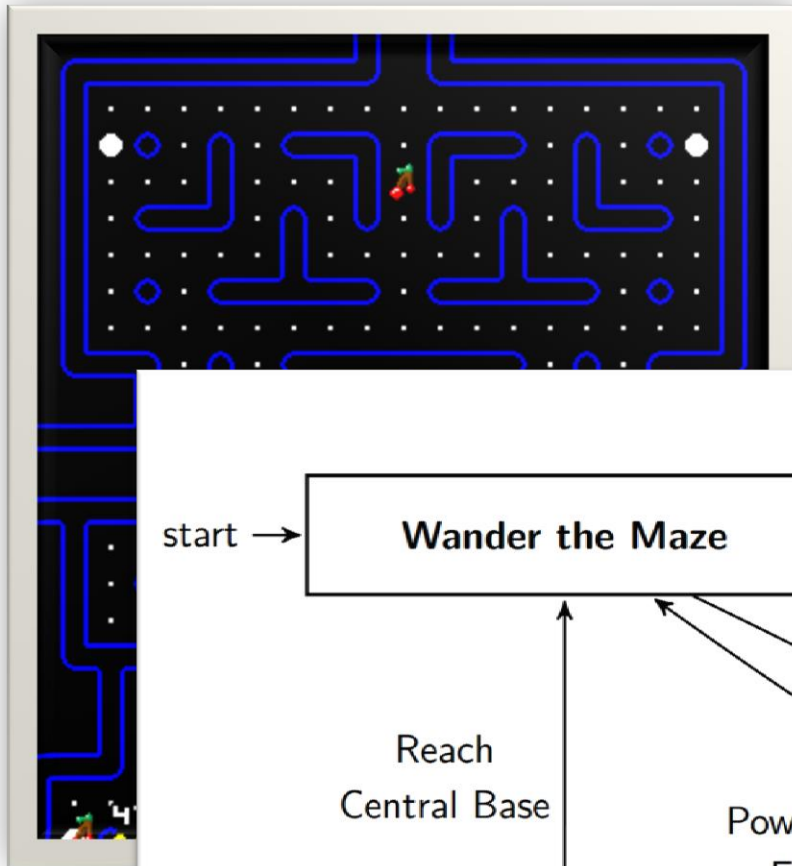


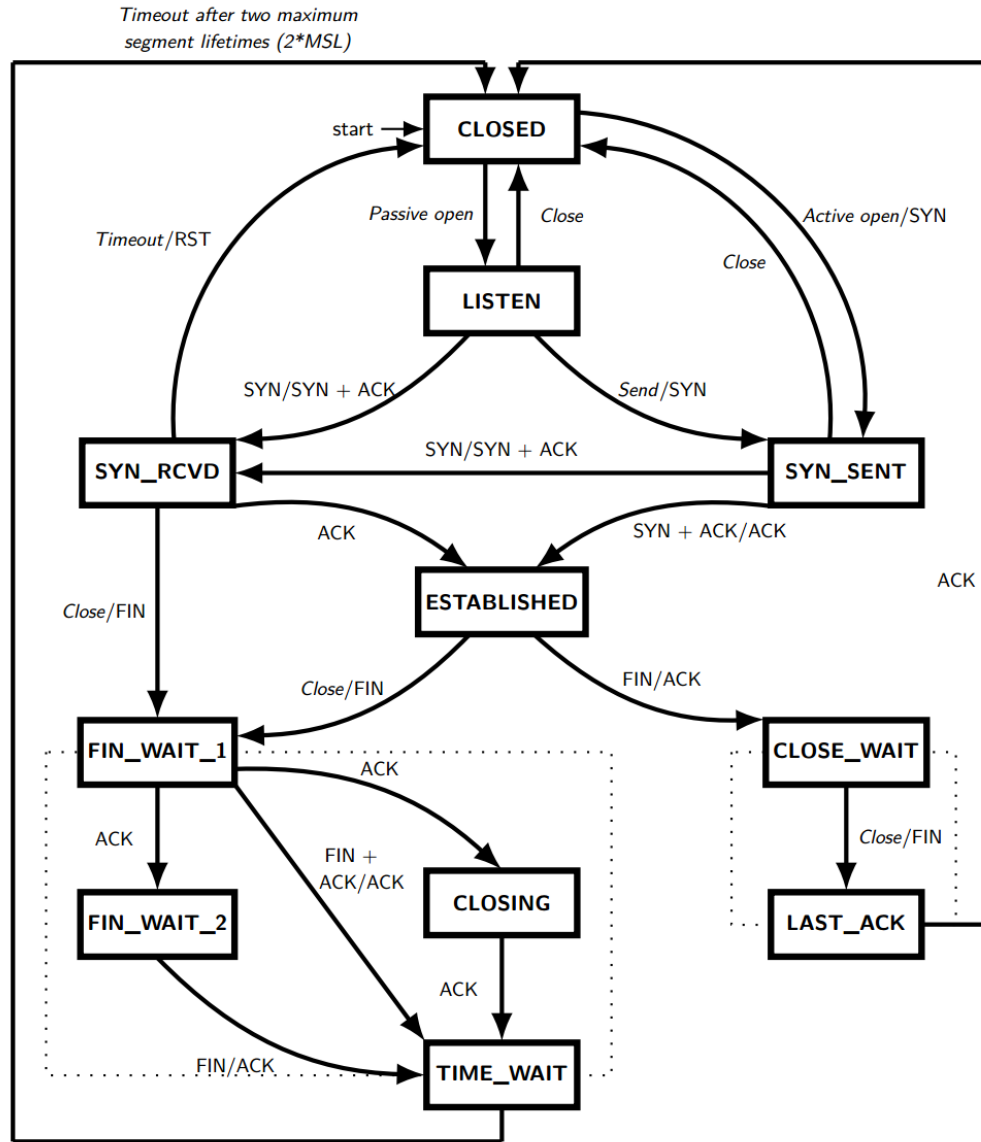
applications of FSMs (aka finite automata)

- Implementation of regular expression matching in programs like `grep`
- Control structures for sequential logic in digital circuits
- Algorithms for communication and cache-coherence protocols
 - Each agent runs its own FSM
- Design specifications for reactive systems
 - Components are communicating FSMs

applications of FSMs (aka finite automata)

- **Formal verification of systems**
 - Is an unsafe state reachable?
- **Computer games**
 - FSMs provide worlds to explore
 - Character AI
- **Minimization algorithms for FSMs can be extended to more general models used in**
 - Text prediction
 - Speech recognition

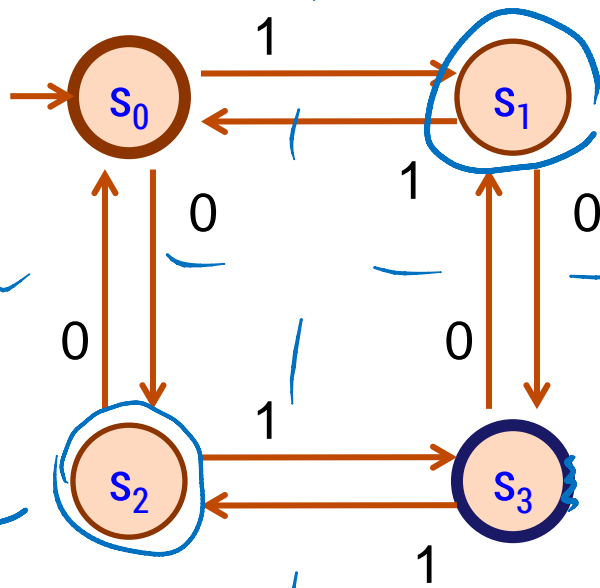




what language does this machine recognize?

$L = \{ \text{strings with odd number of 0s and odd \# of 1s} \}$

1 0 | 1 0 | 1 0

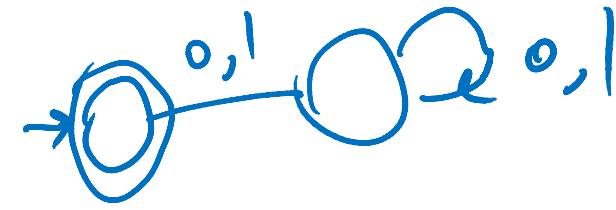
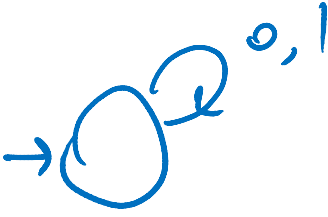
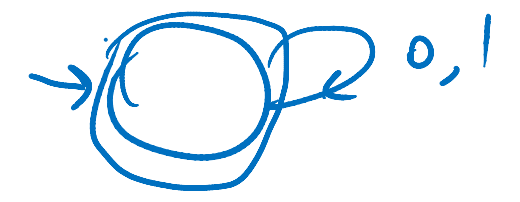



$L_2 = \{ \text{odd \# of 1s, even \# of 0s} \}$

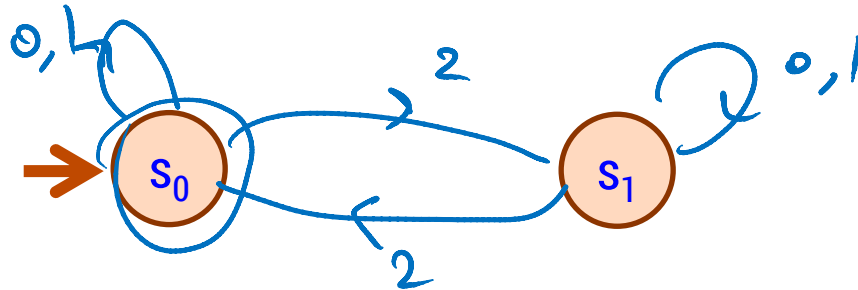
$L = \{ \text{odd \# of 0s, even \# of 1s} \}$

$f(n) = \{ \forall s: \text{len}(s) = n \}$
 $\{ \text{if } \#_0(s) = \text{odd} \}$
 $\#_1(s) = \text{odd} \}$
 $\rightarrow S_3 \}$

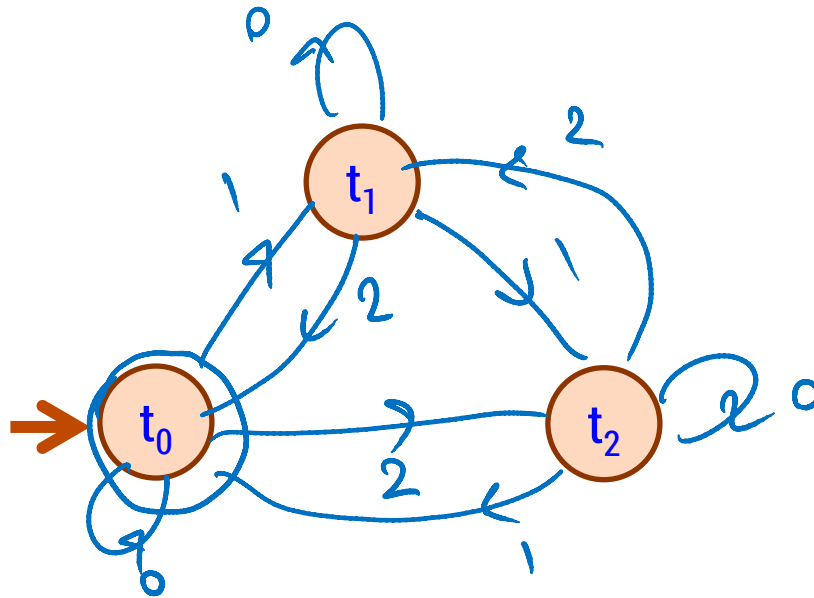
can we recognize these languages with DFAs?

- ϵ A DFA with two states. The start state is a double circle. There is a self-loop on the start state labeled '0,1'. A transition arrow goes from the start state to a second state, which also has a self-loop labeled '0,1'.
- \emptyset A DFA with one state. The start state is a single circle with a self-loop labeled '0,1'.
- Σ^* A DFA with one state. The start state is a double circle with a self-loop labeled '0,1'.
- $\{x \in \{0,1\}^* : \text{len}(x) > 1\}$ A DFA with three states. The start state is a single circle. A transition arrow labeled '0,1' goes to a second state (single circle). Another transition arrow labeled '0,1' goes from the second state to a third state (double circle). The third state has a self-loop labeled '0,1'.

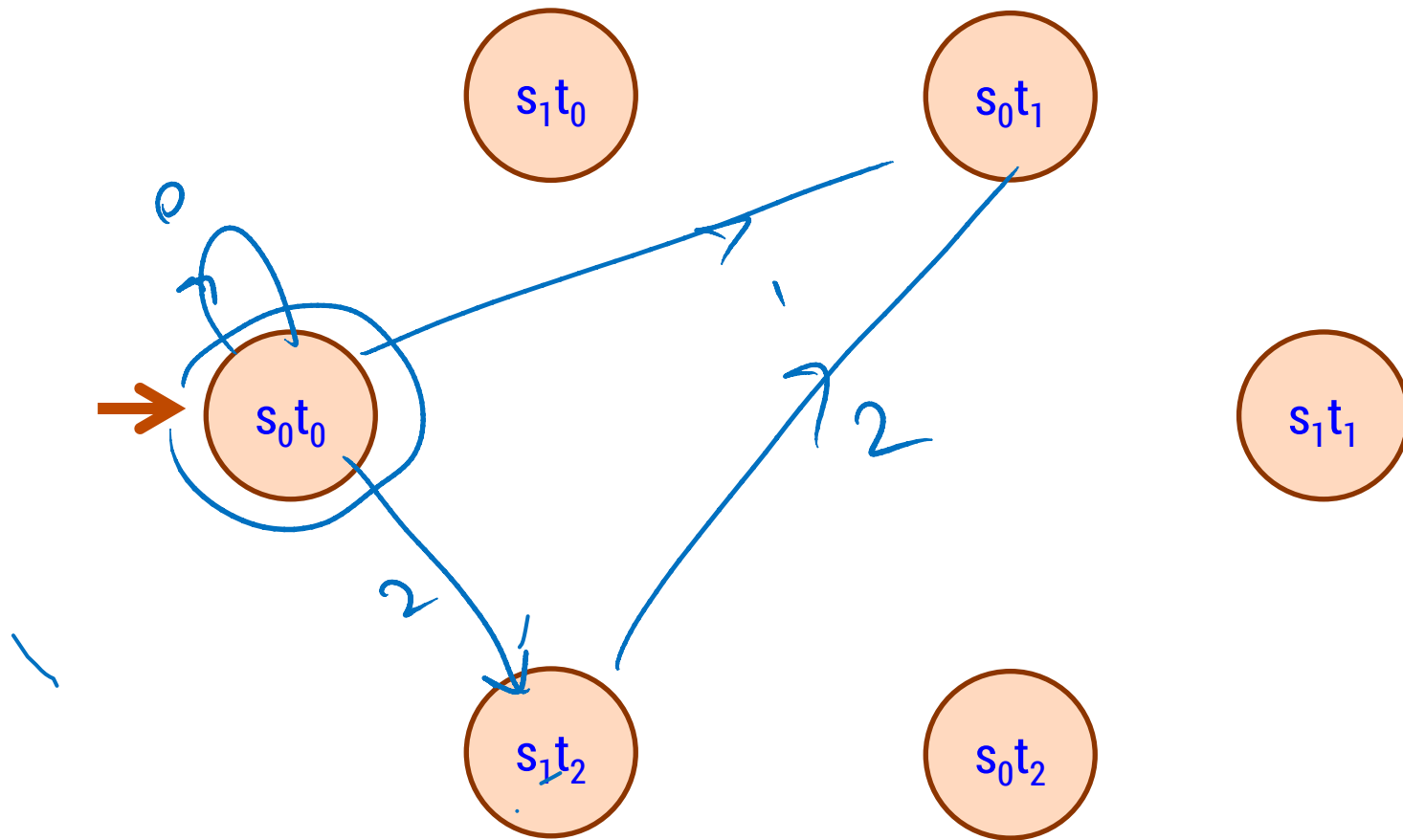
M_1 : Strings with an even number of 2's



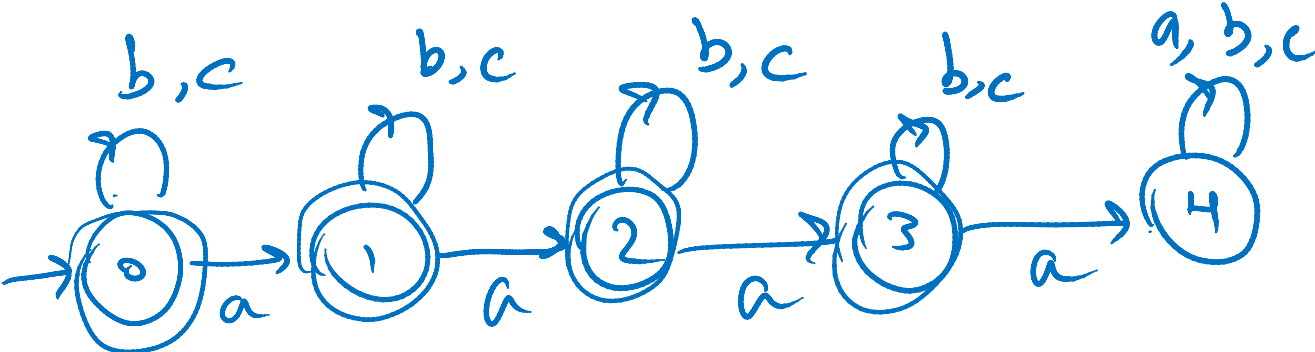
M_2 : Strings where the sum of digits mod 3 is 0



both: even number of 2's and sum mod 3 = 0

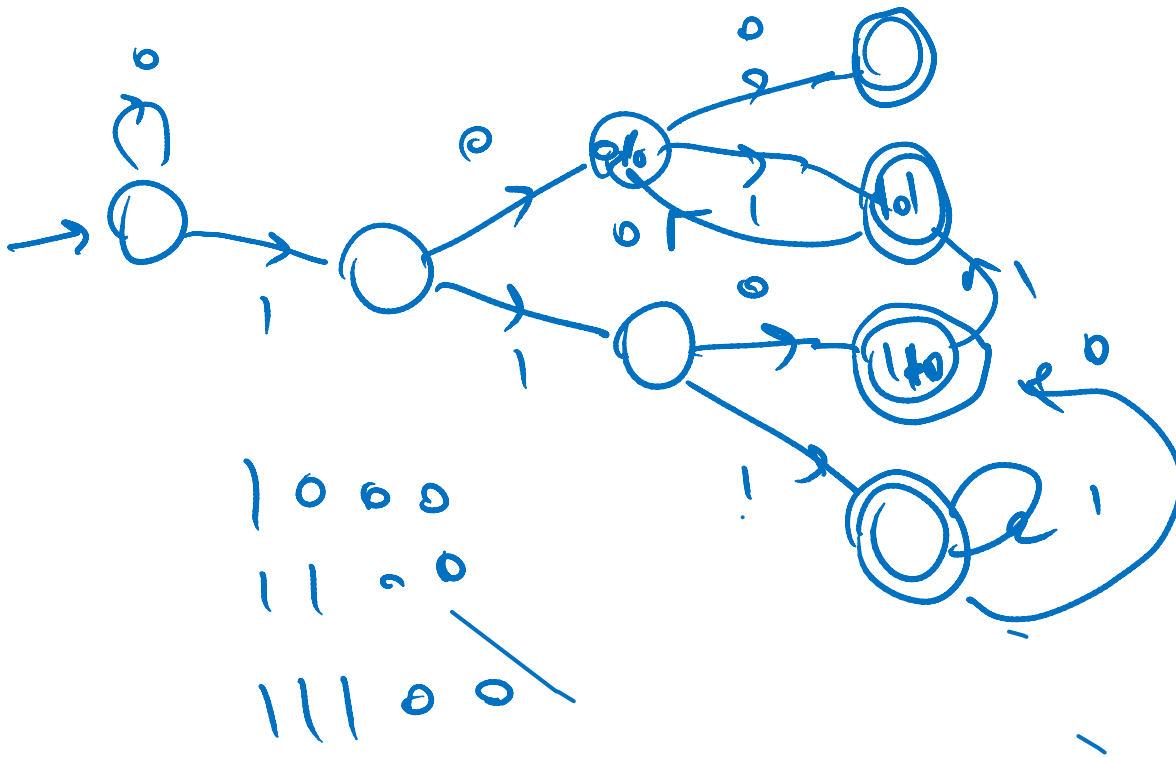


DFA that accepts strings of a's, b's, c's with no more than 3 a's



FSM that accepts binary strings with a 1 three positions from the end

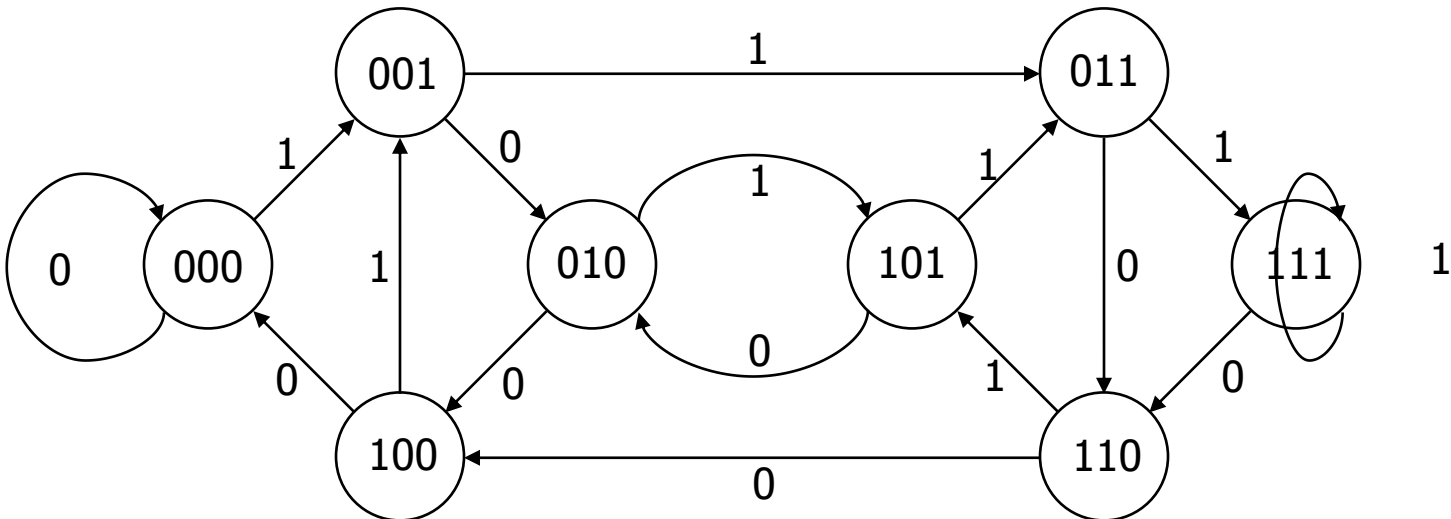
00100 ∈ L
0010 ∉ L
0110 ∈ L

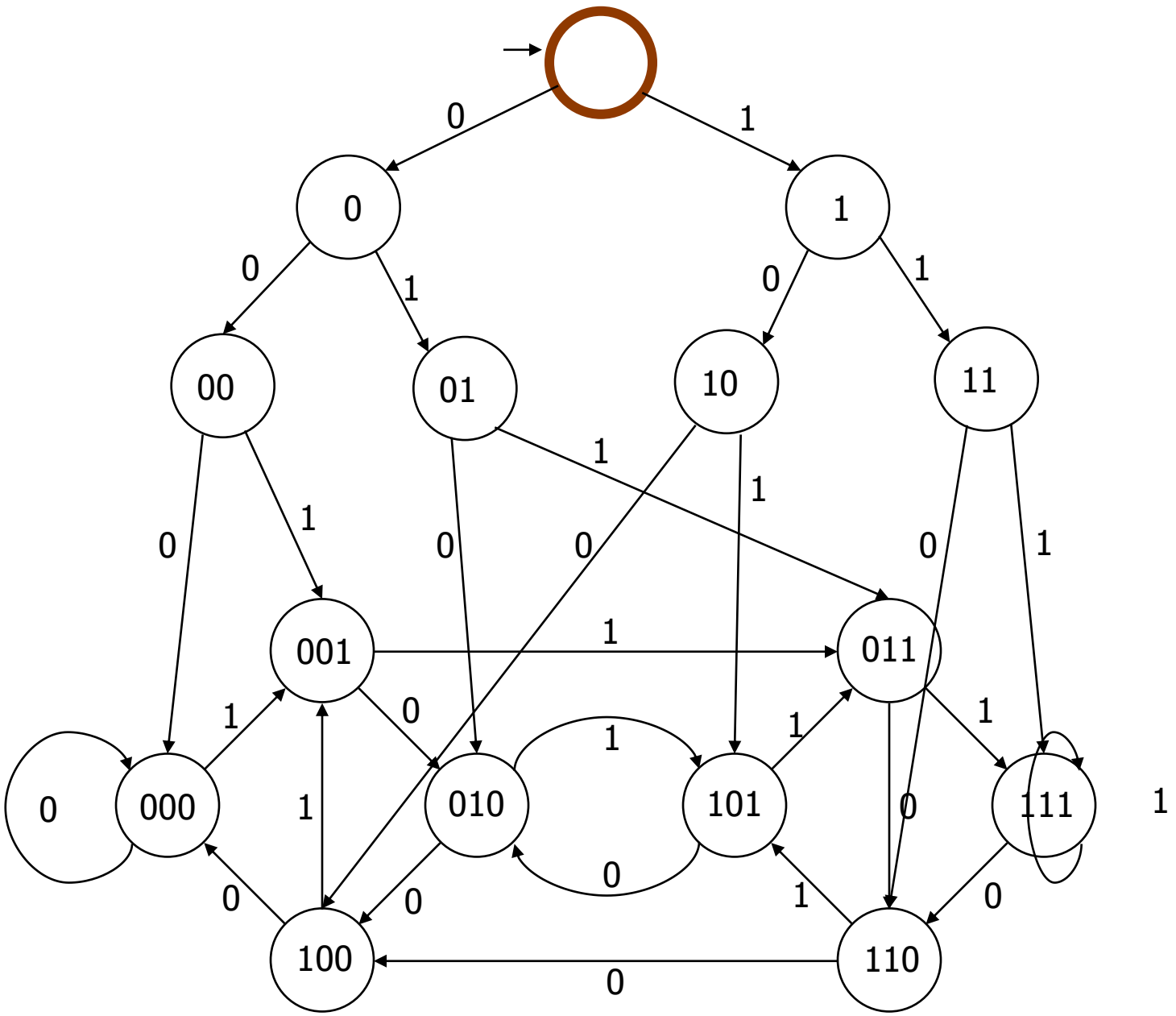


“Remember the last three bits”

3 bit shift register

000 → 001 → 010 → 101 - - -

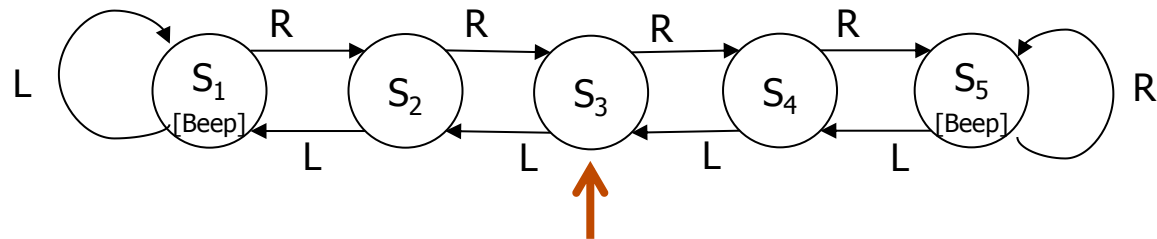




FSMs with output

"Tug-of-war"

State	Input		Output
	L	R	
s_1	s_1	s_2	Beep
s_2	s_1	s_3	
s_3	s_2	s_4	
s_4	s_3	s_5	
s_5	s_4	s_5	Beep





vending machine



We're only making \$5.50/hour writing regular expressions.

Let's design a vending machine.



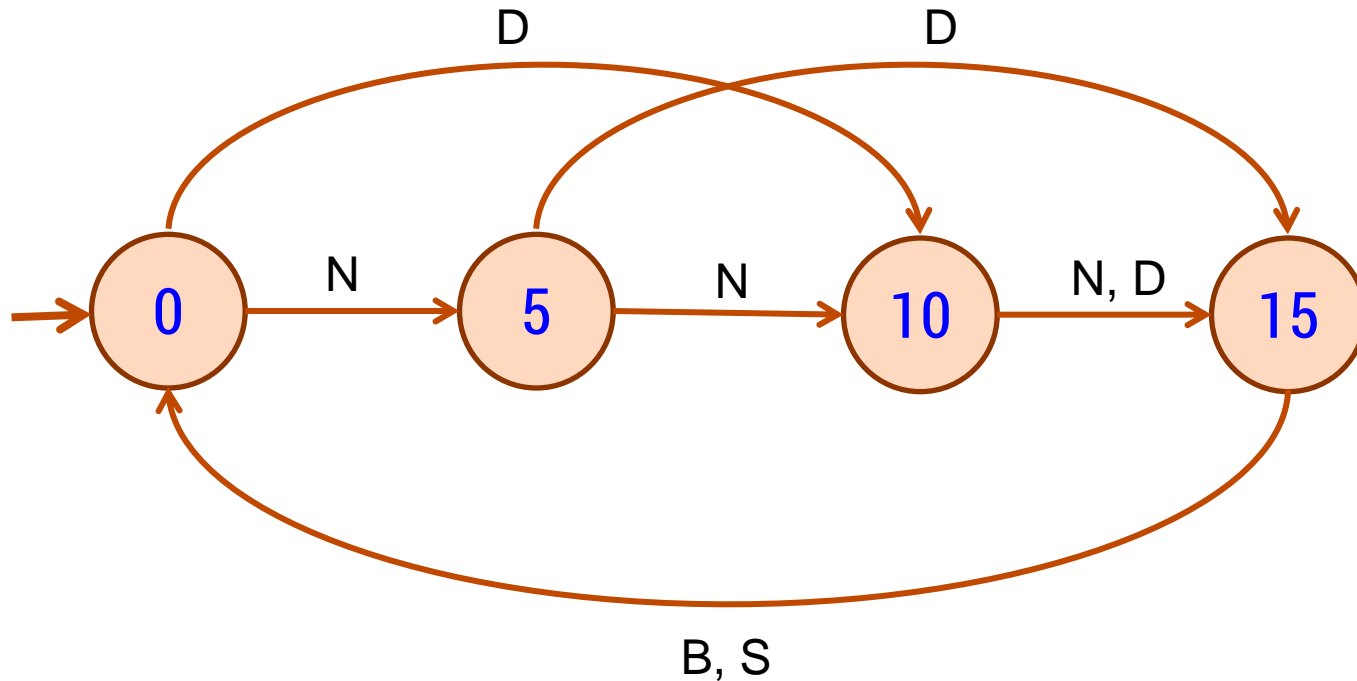
Vending spec:

Enter 15 cents in dimes or nickels

Press **S** or **B** for a candy bar



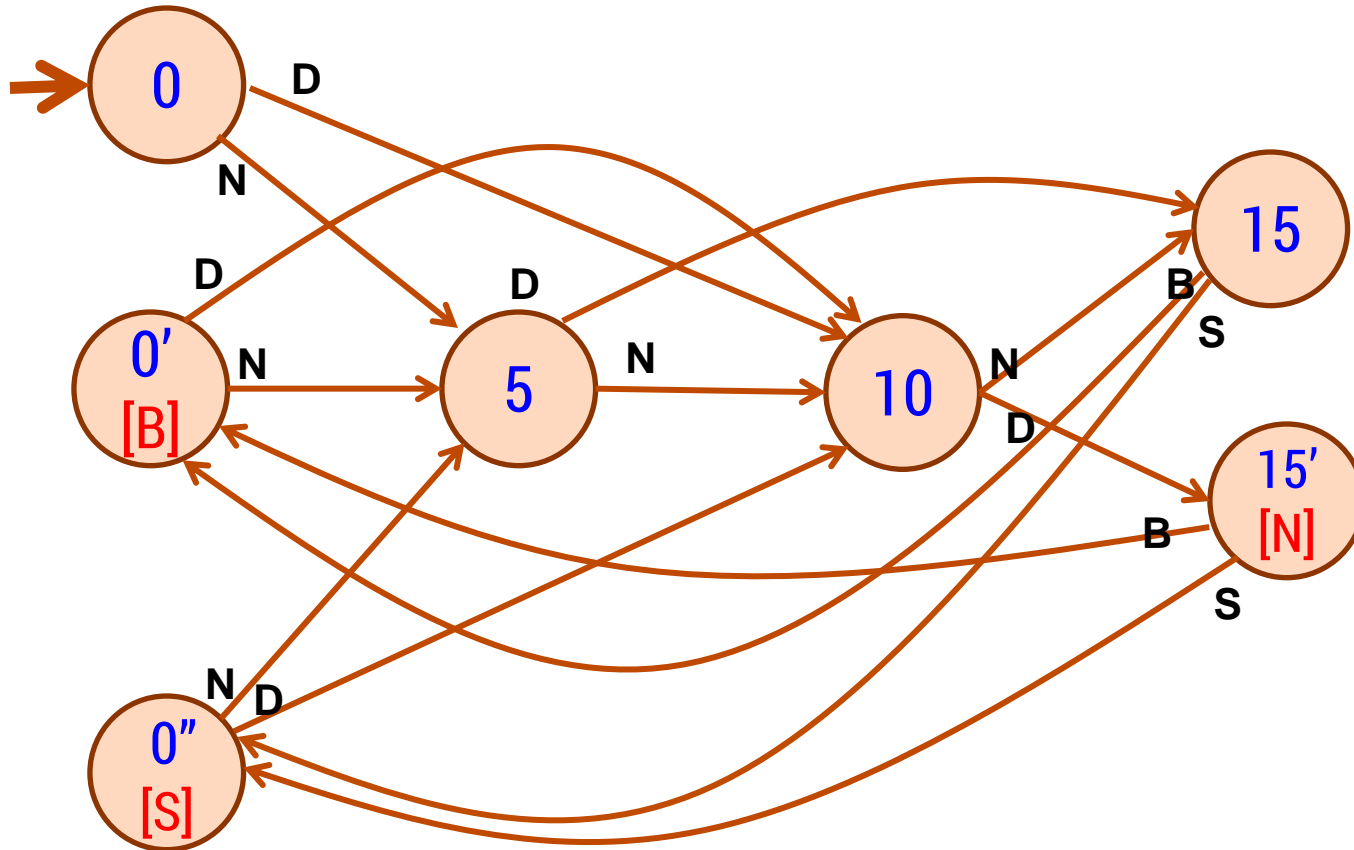
vending machine v0.1



Basic transitions on N (nickel), D (dime), B (butterfinger), S (snickers)



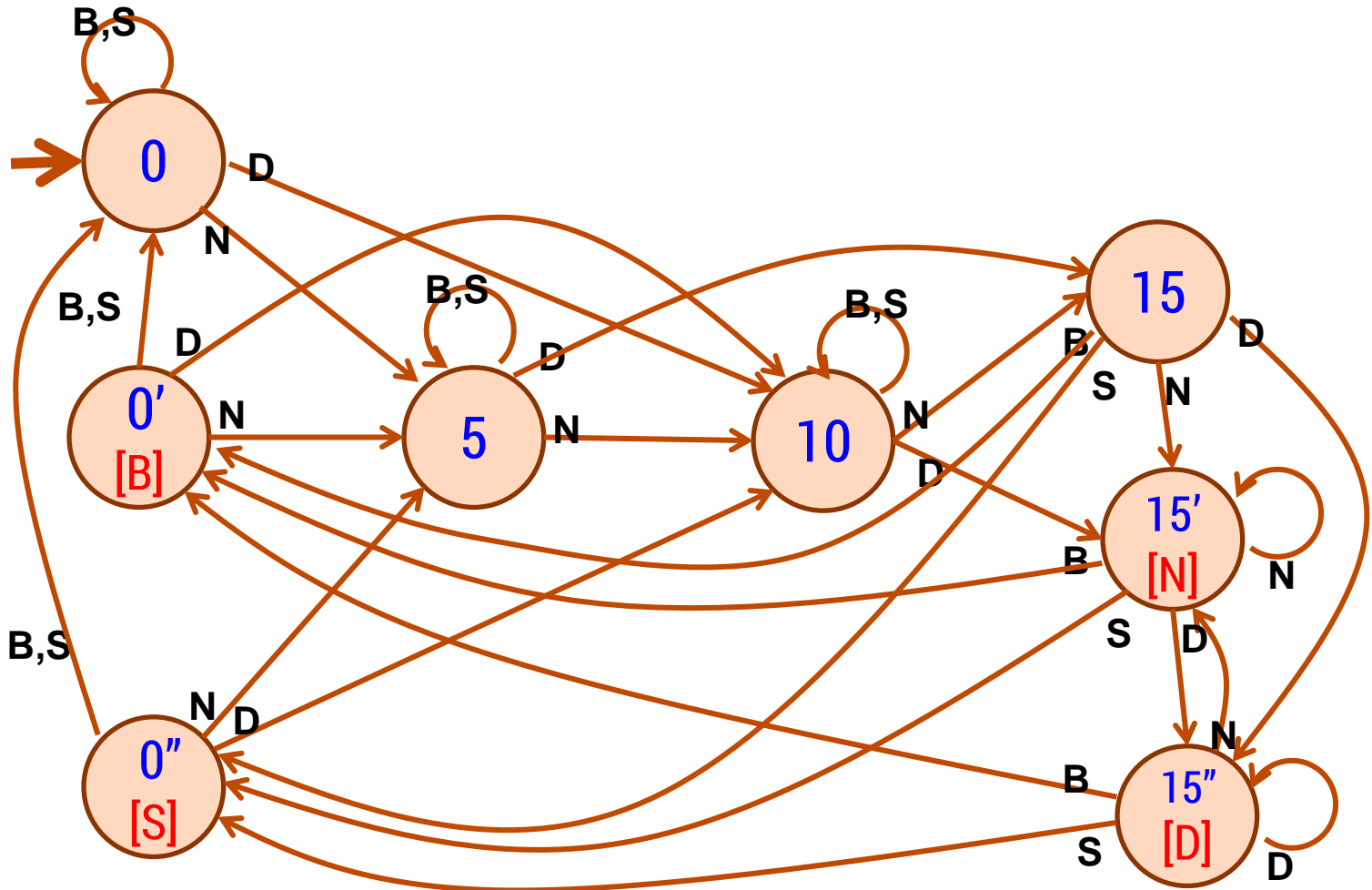
vending machine v0.2



Adding output to states: N – Nickel, S – Snickers, B – Butterfinger



vending machine v1.0



Adding additional "unexpected" transitions