Homework #1 Due Today at 11:59pm

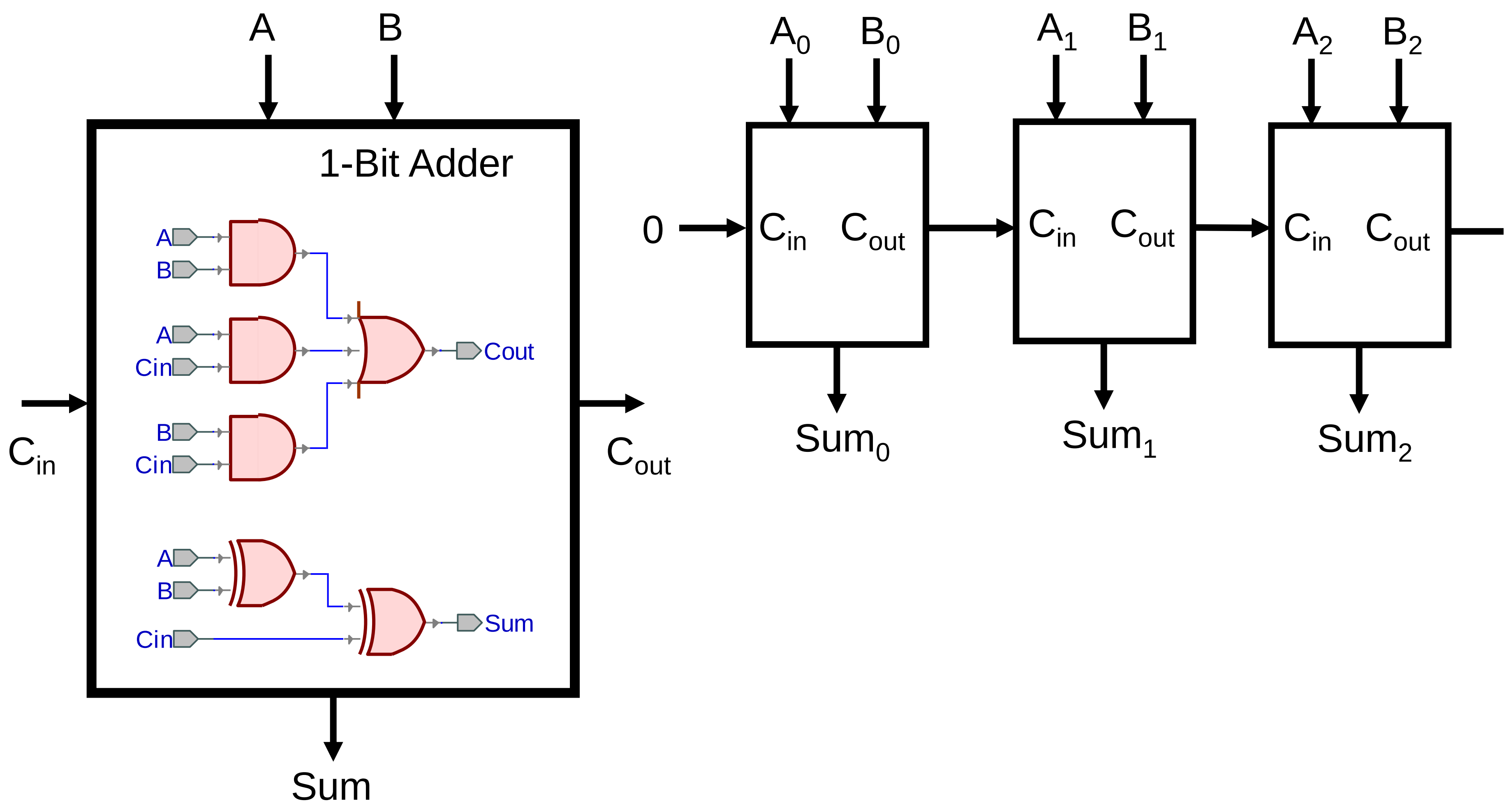
Your Gradescope account is created by your UW/CSE email address

Homework #2 will be posted today and it is due next Friday

TA Office Hours

TA	Office hours	Room
Sam Castle	Wed, 12:00-1:00	CSE 021
Jiechen Chen	Tue, 4:00-5:00	CSE 218
Rebecca Leslie	Mon, 8:30-9:30	CSE 218
Evan McCarty	Tue, 11:30-12:30	CSE 220
Tim Oleskiw	Tue, 3:00-4:00	CSE 218
Spencer Peters	Tue, 1:00-2:00	CSE 218
Robert Weber	Wed, 3:30-4:30	CSE 678 (except Oct 21st at CSE 110)
Ian Zhu	Thu, 4:30-5:30	CSE 021

a 2-bit ripple-carry adder



Fall 2015

Lecture 5: Canonical forms and predicate logic

mapping truth tables to logic gates

Given a truth table:

1. Write the Boolean expression
2. Minimize the Boolean expression
3. Draw as gates
4. Map to available gates

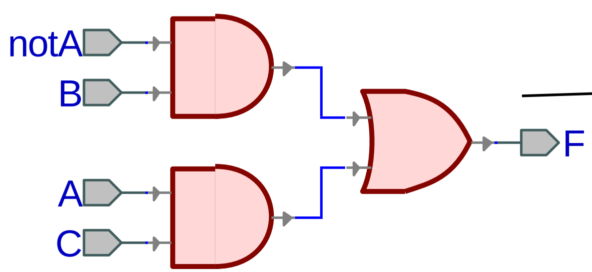
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

1

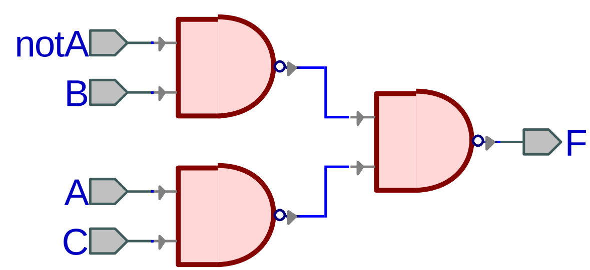
$$F = A'BC' + A'BC + AB'C + ABC$$
$$= A'B(C'+C) + AC(B'+B)$$
$$= A'B + AC$$

2

3



4



- Truth table is the unique signature of a Boolean function
- The same truth table can have many gate realizations
  - we've seen this already
  - depends on how good we are at Boolean simplification
- **Canonical forms**
  - standard forms for a Boolean expression
  - we all come up with the same expression

- also known as Disjunctive Normal Form (DNF)
- also known as minterm expansion

F = *001* *011* *101* *110* *111*

$F = A'B'C + A'BC + AB'C + ABC' + ABC$

A	B	C	F	F′
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Product term (or minterm)

- ANDed product of literals – input combination for which output is true
- each variable appears exactly once, true or inverted (but not both)

A	B	C	minterms
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC

F in canonical form:

$$F(A, B, C) = A'B'C + A'BC + AB'C + ABC' + ABC$$

canonical form ≠ minimal form

$$\begin{aligned} F(A, B, C) &= A'B'C + A'BC + AB'C + ABC + ABC' \\ &= (A'B' + A'B + AB' + AB)C + ABC' \\ &= ((A' + A)(B' + B))C + ABC' \\ &= C + ABC' \\ &= ABC' + C \\ &= AB + C \end{aligned}$$

- Also known as Conjunctive Normal Form (CNF)
- Also known as maxterm expansion

F = *000* *010* *100*

F = (A + B + C) (A + B′ + C) (A′ + B + C)

A	B	C	F	F′
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Complement of function in sum-of-products form:

- $F' = A'B'C' + A'BC' + AB'C'$

Complement again and apply de Morgan’s and get the product-of-sums form:

- $(F')' = (A'B'C' + A'BC' + AB'C')'$
- $F = (A + B + C)\,(A + B' + C)\,(A' + B + C)$

A	B	C	F	F'
0	0	0	0	1
			1	0
0	1	0	0	1
			1	0
1	0	0	0	1
			1	0
			1	0
			1	0

product-of-sums canonical form

Sum term (or maxterm)

- ORed sum of literals – input combination for which output is false
- each variable appears exactly once, true or inverted (but not both)

A	B	C	maxterms	F
0	0	0	A+B+C	1
0	0	1	A+B+C′	0
0	1	0	A+B′+C	1
0	1	1	A+B′+C′	1
1	0	0	A′+B+C	1
1	0	1	A′+B+C′	1
1	1	0	A′+B′+C	1
1	1	1	A′+B′+C′	0

F in canonical form:

$F(A, B, C) = (A + B + C)\,(A + B' + C)\,(A' + B + C)$

$F = (A + B + C')\,(A' + B' + C')$

canonical form ≠ minimal form

$$
\begin{aligned}
F(A, B, C) &= (A + B + C)\,(A + B' + C)\,(A' + B + C) \\
&= (A + B + C)\,(A + B' + C) \\
&\quad\ (A + B + C)\,(A' + B + C) \\
&= (A + C)\,(B + C)
\end{aligned}
$$

- Propositional Logic
  - If Pikachu doesn't wear pants, then he flies on Bieber's jet unless Taylor is feeling lonely.
- Predicate Logic
  - If x, y, and z are positive integers, then $x^3 + y^3 \neq z^3$.

Predicate or Propositional Function

- A function that returns a truth value, e.g.,

  "x is a cat"

  "x is prime"

  "student x has taken course y"

  "$x > y$"

  "$x + y = z$" or Sum(x, y, z)

  "$5 < x$"

Predicates will have variables or constants as arguments.

We must specify a “domain of discourse”, which is the possible things we’re talking about.

“x is a cat”
(e.g., mammals)

“x is prime”
(e.g., positive whole numbers)

student x has taken course y”
(e.g., students and courses)

$\forall x\ P(x)$

P(x) is true for every x in the domain

read as "for all x, P of x"

$\exists x\ P(x)$

There is an x in the domain for which P(x) is true

read as "there exists x, P of x"

statements with quantifiers

- $\exists x\ \mathrm{Even}(x)$ — 2 — T
- $\forall x\ \mathrm{Odd}(x)$ — 2 — F
- $\forall x\ (\mathrm{Even}(x) \vee \mathrm{Odd}(x))$ — T
- $\exists x\ (\mathrm{Even}(x) \wedge \mathrm{Odd}(x))$ — 2 — F
- $\forall x\ \mathrm{Greater}(x+1, x)$ — T
- $\exists x\ (\mathrm{Even}(x) \wedge \mathrm{Prime}(x))$ — 2 — T

Domain:
Positive Integers

Even(x)
Odd(x)
Prime(x)
Greater(x,y)
(or “x>y”)
Equal(x,y)
(or “x=y”)
Sum(x,y,z)
(or “z=x+y”)

2 10

- $\forall x\, \exists y$ Greater (y, x) T

- $\forall x\, \exists y$ Greater (x, y) F

  $x = 1$

- $\forall x\, \exists y$ (Greater$(y, x) \wedge$ Prime(y)) T

  $x = 20$ $y = 23$

- $\forall x$ (Prime$(x) \rightarrow$ (Equal$(x, 2) \vee$ Odd(x))) T

- $\exists x\, \exists y$ (Sum$(x, 2, y) \wedge$ Prime$(x) \wedge$ Prime(y)) T

  $x = 3$ $y = 5$

Domain:
Positive Integers

Even(x)
Odd(x)
Prime(x)
Greater(x,y)
(or "x>y")
Equal(x,y)
(or "x=y")
Sum(x,y,z)
(or "z=x+y")

	Prev	Now
• $\forall x\ \exists y$ Greater (y, x)	T	T
• $\forall x\ \exists y$ Greater (x, y)	F pos int	T

Domain:
All integers

Even(x)
Odd(x)
Prime(x)
Greater(x,y)
(or "x>y")
Equal(x,y)
(or "x=y")
Sum(x,y,z)
(or "z=x+y")

Domain of quantifiers is important!

- "Red cats like tofu"

Cat(x)
Red(x)
LikesTofu(x)

$\forall x\ ((Cat(x) \wedge Red(x)) \to LikesTof(x))$ Domain mammals

- "Some red cats don't like tofu"

$\exists x\ (Cat(x) \wedge Red\, x) \cancel{\to} \neg\, likeTofu(x)$

$\wedge$

Domain
Pos Int

- not every positive integer is prime

$\neg \forall x \; prime(x)$

- some positive integer is not prime

$\exists x \; \neg Prime(x)$

- prime numbers do not exist

$\neg \exists x \; prime(x)$

- every positive integer is not prime

$\forall x \; \neg Prime(x)$

Domain:
Fruit

PurpleFruit(x)

$\forall x$ PurpleFruit(x)

Every fruit is purple

Which one is equal to $\neg\forall x$ PurpleFruit(x)?

- $\exists x$ PurpleFruit(x)?

  There is a purple fruit

- $\exists x$ $\neg$PurpleFruit(x)?

  There is at least one fruit which is not purple.

$$\boxed{\begin{aligned} \neg\forall x\ P(x) &\equiv \exists x\,\neg P(x) \\ \neg\exists x\ P(x) &\equiv \forall x\,\neg P(x) \end{aligned}}$$

$$\boxed{\begin{array}{l} \neg\forall x\ P(x) \equiv \exists x\ \neg P(x) \\ \neg\exists x\ P(x) \equiv \forall x\ \neg P(x) \end{array}}$$

“There is no largest integer.”

$$\begin{aligned} & \neg\exists x\ \forall y\ (x \geq y) \\ \equiv\ & \forall x\ \neg\forall y\ (x \geq y) \\ \equiv\ & \forall x\ \exists y\ \neg(x \geq y) \\ \equiv\ & \forall x\ \exists y\ (y > x) \end{aligned}$$

“For every integer there is a larger integer.”

example: Notlargest(x) $\equiv \exists\, y$ Greater (y, x)
$\equiv \exists\, z$ Greater (z, x)

truth value:

doesn't depend on y or z "bound variables"

does depend on x "free variable"

quantifiers only act on free variables of the formula they quantify

$$\forall x\, (\exists y\, (P(x, y) \rightarrow \forall x\, Q(y, x)))$$

$\exists x\ (P(x) \land Q(x))$ vs. $\exists x\, P(x) \land \exists x\, Q(x)$