CSE 311 Sample Final Exam Problems, Fall 4013

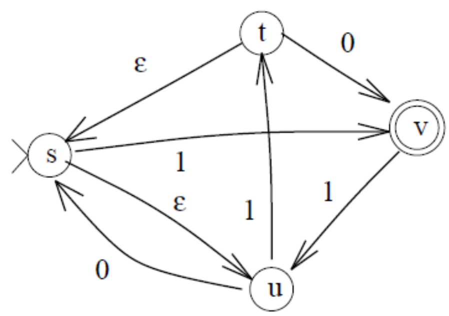
All the practice midterm problems plus:

Problem 1:

2. (a) Draw the state diagram of an NFA M that recognizes the language $a^*b(b \cup ab)^*a$. You don't have to use any particular construction method but you should try to avoid unnecessary states.

Problem 2:

3. Build a DFA equivalent to the following NFA using the "subset construction." You only need to show states that are reachable from the start state of your DFA (*but* do not attempt to simplify further).



Problem 3:

2. (30 points) Define the language $A = \{w \in \{0,1\}^* \mid$ the number of 0's minus the number of 1's in w is divisible by $3\}$.

   (a) Construct a DFA with only three states that recognizes A.

Problem 4:

Show that the following language cannot be recognized by a DFA:

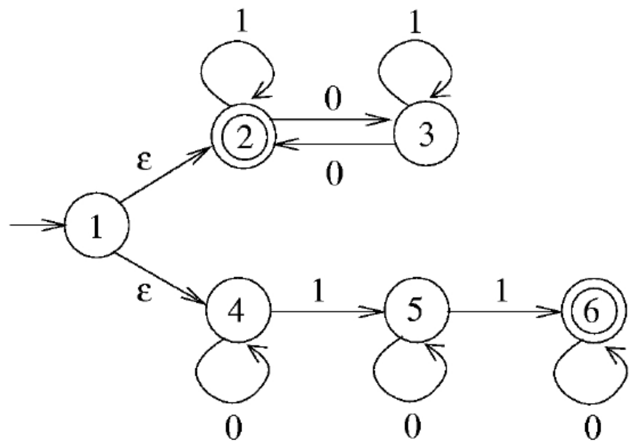
$$\{a^n b a^m b a^{m+n} \mid n, m \geq 1\}$$

Give a CFG for the language:

$$\{a^n b a^m b a^{m+n} \mid n, m \geq 1\}$$

Problem 5:

Consider the NFA N = (Q, Σ, δ, q_0, F) with the following state diagram:



a. What states can N be in after reading:
   the string 0? __________ the string 01? __________ the string 0111? _________
b. Does N accept 0111? _____ Why or why not?

Problem 6:

Let L be the set of strings in {a, b}* such that each a, if any, has two b's immediately to its right. Give:

a) A finite automaton accepting L.
b) A regular expression denoting L.
c) A Context free grammar generating L.

Problem 7:

In the land of Garbanzo, the unit of currency is the bean. They only have two coins, one worth 2 beans and the other worth 5 beans.

(a) Give a recursive definition of the set of positive integers S such that x is in S if and only if one can make up an amount worth x beans using at most one 5-bean coin and any number of 2-bean coins.
(b) Prove by strong induction that every integer ≥ 4 is in S.

Problem 8:

Let R be the relation { (1,2), (3,4), (1,3), (2,1) } defined on the set {1,2,3,4,5}.

(a) Draw the graph of R.
(b) Draw the graph of the R^2.
(c) Draw the graph of the reflexive-transitive closure of R.

Problem 9:

For each n≥ 0 define T_n, the "complete 3-ary tree of height n" as follows:

- T_0 is an undirected graph consisting of a single vertex called the *root* of T_0.
- For n≥0, T_{n+1} is an undirected graph consisting of a new vertex of degree 3 joined to the roots of 3 disjoint copies of T_n. The new vertex is called the root of T_{n+1}.

Prove by induction that for each n≥ 0, T_n has exactly $(3^{n+1}-1)/2$ vertices.

Problem 10:

Suppose R_1 and R_2 are transitive-reflexive relations on a set A. Is the relation $R_1 \cup R_2$ necessarily a transitive-reflexive relation? Justify your answer.

Problem 11:

- Every day, starting on day 0, one vampire arrives in Seattle from Transylvania and, starting on the day after its arrival, bites one Seattlite every day. People bitten become vampires themselves and live forever. New vampires also bite one person each day starting the next day after they were bitten. Let V_n be the number of vampires in Seattle on day n. So, for example, $V_0 = 1$, $V_1 = 3$ (one that arrived from Transylvania on day 0, one that he bit on day 1, and another one that arrived from Transylvania on day 1), $V_2 = 7$ and so on. Write a recurrence relation for V_n that is valid for any $n \geq 2$.
- Prove by induction on n that $V_n \leq 3^n$.

Problem 12:

Suppose that for all $n \geq 1$

$$g(n+1) = max_{1 \leq k \leq n}[g(k) + g(n+1-k) + 1]$$

and that $g(1) = 0$. Prove by induction that $g(n) = n - 1$ for all $n \geq 1$.

Problem 13:

Circle T or F to indicate whether each of the following statements is True or False. Also, briefly JUSTIFY each answer.

There is a program that takes as input the source code $\langle R \rangle$ of a program together with an input x for R, and will output 1 if R halts on input x and will not output 1 if R does not halt on input x. . . . . T F

There is a program that always halts that, given the source code $\langle Q \rangle$ of a Java program, can tell whether or not the program $\langle Q \rangle$ actually reads any input. T F

Problem 14:

Design a circuit with four inputs a_0, a_1, a_2, a_3, and two outputs b_0, b_1, where $b_1 b_0$ give the binary representation of the index of a true input *if exactly one input is true*. If no inputs are true, or more than one input is true the outputs are undefined (meaning we don't care what values they take on).

a) Give a table of values for this circuit as a function of its inputs. You only need to provide the rows for input combinations that have exactly one input that is true.

b) Express the outputs as functions of the inputs using boolean algebra notation for the logical operators.

c) Draw the circuit for your solution using the gates AND, OR, and NOT. (If you don't remember the gate shapes it is OK to just write AND, OR, NOT in them.)

Problem 15:

Use the algorithm for minimizing DFA's to find a minimal DFA equivalent to the following DFA. Show your work including the results of, and reasons for, each step of this execution.