CSE 311 Quiz Section 7: May 15, 2014

1. Midterm exam: solutions to #3 and #5
2. Definition of a full binary tree:
   (a) BASIS: There a binary tree with a single node. That node is the root of the tree.
   (b) RECURRENCE: Two disjoint full binary trees T_1 and T_2 can be used to form a new full binary tree, as follows. Create a new node as the root. Use two edges to join that root with the roots of T_1 and T_2.

   - What is the difference between a full binary tree and an extended binary tree (the subject of Homework 6, exercise 1)?
   - Prove that every full binary tree with k leaves has $k-1$ nonleaf nodes.

3. Define the Fibonnaci numbers as follows: $f(0) = 0, f(1) = 1$, and $f(n) = f(n-2) + f(n-1)$ for all integers $n > 1$. Prove by induction that, for all nonnegative integers n, the number of iterations used by Euclid's algorithm to compute $gcd(f(n+1), f(n))$ is n.

   Proof: The basis is $n = 0$, and indeed $gcd(1, 0)$ uses no iterations. For the induction step, the first iteration changes the arguments from $(f(n+1), f(n))$ to $(f(n), f(n-1))$, and the induction hypothesis says it takes $n-1$ more iterations to finish the computation.

   The only hitch is that the theorem is false for almost all values of n. For your entertainment, find the flaw in the proof. (It's not hard to find once you know it's false, but I find the proof absolutely convincing if you don't suspect it's false.)