CSE 311 Quiz Section 6: May 8, 2014

1. Example of a subtle error in a proof by induction:

   "All horses are the same color."

   You can find a pseudo-proof and an explanation in the wikipedia web page:
   `http://en.wikipedia.org/wiki/All_horses_are_the_same_color`

2. Prove the following using induction (from 7th ed. p.318):

$$\sum_{j=0}^{n} ar^j = \frac{ar^{n+1} - a}{r-1} \qquad \text{when } r \neq 1,$$

   where n is a nonnegative integer.

3. "Define the Fibonnaci numbers as follows: $f(0) = 0, f(1) = 1$, and $f(n) = f(n-2) + f(n-1)$ for all integers $n > 1$. Prove by induction that, for all nonnegative integers n, the number of iterations used by Euclid's algorithm to compute $gcd(f(n+1), f(n))$ is n."

   Proof: The basis is $n = 0$, and indeed $gcd(1, 0)$ uses no iterations. For the induction step, the first iteration changes the arguments from $(f(n+1), f(n))$ to $(f(n), f(n-1))$, and the induction hypothesis says it takes $n - 1$ more iterations to finish the computation.

   The only hitch is that the theorem is false for almost all values of n. For your entertainment, find the flaw in the proof. (It's not hard to find once you know it's false, but I find the proof absolutely convincing if you don't suspect it's false.)

4. Prove the following:

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + ... + \frac{1}{n^2} \leq 2 \ , \ n \geq 1$$

   Hint1: Try replacing the right hand side of the inequality with something that will make the statement stronger.

   Hint2: Ask the TA.