CSE 311 Quiz Section 10: June 4, 2014
Final Exam Review

1. Show that $\neg(p \leftrightarrow q) \equiv (p \ \wedge \neg q) \vee (\neg p \wedge q)$.
2. Answer with True or False and explain:
   (a) $(p \vee q) \rightarrow p$ is a tautology.
   (b) $\neg p \rightarrow (p \rightarrow q)$ is a tautology.
   (c) $11 \equiv 19 \ (mod\ 4)$
3. Find the sum-of-products expansion of the Boolean function $F(w, x, y, z)$ that has the value 1 if and only if an odd number of w, x, y, z have value 1.
4. Show that $A \subseteq B \leftrightarrow \bar{B} \subseteq \bar{A}$.
5. Construct circuits from inverters, AND gates, and OR gates to produce these outputs.
   (a) $\bar{x} + y$
   (b) $xyz + \bar{x}y$
   (c) $(\overline{x + y})(\overline{y + z})(\overline{x + z})$
6. Design a circuit that implements majority voting for five individuals.
7. Consider the following relations on the set $\{1, 2, 3, 4\}$:
   (a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
   (b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
   (c) $\{(2, 4), (4, 2)\}$

   For each one of them decide whether it is: reflexive, symmetric, antisymmetric, transitive.
8. Give logical expressions for the following statements. Use quantifiers, connectives, and the predicates $P(x)$ and $H(x)$ which mean "x passed the class" and "x turned in all of the homework".
   (a) Every student that passed the class turned in all of the homework.
   (b) There was a student that passed the class, but did not turn in all of the homework.
9. Let $L = \{w \in \{0, 1\}^*\}$ where the number of zeroes minus the number of ones in w is divisible by 3. Construct a DFA wih only 3 states that recognizes L.

10. In the land of Garbanzo, the unit of currency is the bean. They only have two coins, one worth 2 beans and the other worth 5 beans. Give a recursive definition of the set of positive integers S such that x is in S if and only if one can make up an amount worth x beans using at most one 5-bean coin and any number of 2-bean coins.

11. Suppose that R_1, R_2 are transitive and reflexive relations on a set A. Is their union transitive and reflexive? Explain your answer.

12. Every day, starting on day 0, one vampire arrives in Seattle from Transylvania. Starting on the day of its arrival, he bites one Seattlite every day. People bitten become vampires themselves and live forever. New vampires also bite one person each day starting the next day after they were bitten. Let V_n be the number of vampires in Seattle on day n. So, for example, $V_0 = 1, V_1 = 3$ (one that arrived from Transylvania on day 0, one that he bit on day 1 and another one that arrived from Transylvania on day 1),$V_2 = 7$ and so on.

    (a) Write a recurrence relation for V_n that is valid for any $n \geq 2$.

    (b) Prove by induction on n that $V_n \leq 3^n$.